NEXUS BETWEEN RENEWABLE & NON-RENEWABLE ENERGY CONSUMPTION AND ECONOMIC GROWTH IN PAKISTAN: A REVISIT USING MAXIMUM ENTROPY BOOTSTRAP ANALYSIS

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ABSTRACT
This paper investigated the causal directions of disaggregated energy consumption (renewable & non-renewable energy consumption) with economic growth by employing the new state of art technique called as maximum entropy bootstrap method (meboost) for Pakistan during 1975-2021. Since such technique is robust in the sense that unlike the existing methodologies, this approach has no reliance on the conventional assumptions of asymptotic theory enabling meboost to robust inferences with small size. The method is also independent of specification issues, lag length selection, and can be applied even in the presence of any kind of structural gap and non-stationarity of dataset without manipulation of dataset for stationarity. The observed results supported the growth-hypothesis for renewable energy model and concluded conservation hypothesis for non-renewable energy consumption model. In lights of obtained results, it is recommended that Pakistan's government should proactively develop conservation policies for non-renewable energy consumption and implementation policies for renewable energy consumption through different sources including domestic energy resources, hydropower resources and solar energy.
Key Words: Renewable & Non-Renewable Energy, Causality, Economic Growth, Maximum Entropy Bootstrapping Method.

INTRODUCTION
In recent years, one of the most pressing concerns for public policy has been the maintenance of a healthy equilibrium between economic expansion and environmental preservation. Policies are being put forth with the goal of reducing dependency on non-renewable energy resources, guaranteeing energy security, and alleviating poverty. These policies attempt to bring about a cleaner environment without harming economic growth rates. In the recently held COP-26 conference in United Kingdom, it is aimed to prioritize implementation among the member countries towards adoption of sustainable energy. During the COP26 conference, about 39 signatories have committed to transform conventional energy into clear energy which has worth of $28bnt. Similarly UK opens door of business for renewable energy valued up $7.5bn into 2024. Likewise, many countries including Pakistan, India, China etc. has also committed towards consumption of renewable energy. However, it may be interesting to know that transformation of fossil fuel energy into renewable energy has any impact on economic growth? Hence the relationship between economic growth (thereafter "ECGT") and energy consumption (thereafter "ECGT") remains one of the most talked obout subjects in the existing literature (Kaplan et al., 2011; Apergis and Payne, 2010; Irfan et al., 2011). Energy consumption plays a crucial role in economic growth through various economic activities, such as production, transportation,

and on consumption of goods. The goal of sustainable development is to reduce both energy consumption and economic growth. Studies on ENGY and ECGT have become more appealing as a result of international agreements to save energy and cut CO2 emissions. The use of renewable energy sources, however, is the main factor in those researches. As the importance of sustainable development grows, renewable energy sources are now viewed as one of the most essential segments of the combined consumption of energy in the globe. Researchers have shown more inclination on analyzing the impact of adoptability of renewable energy on ECGT. Concerns about the volatility of the oil price, our reliance on foreign energy supplies, and carbon emissions are the main drivers of the current interest in renewable energy sources. Government initiatives such as tax crediting to industries for producing green and clean energy, reimbursements for the solar energy and issuance of markets certificates for renewable energy consumption (thereafter "RNENGY") have aided in the adoptability of renewable energy as a practical module of the energy portfolios of numerous nations (Bowden and Payne, 2010). In contemporary years, investors are more inclined for their investment in renewable and sustainable energy particularly in the areas of solar and wind power. This has led to improvements in the efficiency and cost effectiveness of renewable energy systems, making them more competitive with traditional energy sources. The uses of energy may have many impacts on economic variables; however economic growth is more vulnerable to consumption of either RNENGY or non-renewable energy consumption (thereafter "NRNENGY").

Since many of the studies are available on the functional relationship between ENGY and ECGT (see Apergis and Payne, 2010b, 2015; Atulkar and Kesari, 2014; Huang, et al., 2008; Kaplan et al., 2011; Lise and Montfort, 2007; Soytas and Sari, 2009 for details). Numerous scholars have examined the relationship between RNENGY and ECGT in the available literature. These studies states that RNENGY may boost the economy by generating new employment, cutting energy prices, and boosting activity in rural and isolated areas. Additionally, lowering reliance on foreign imports of fossil fuels can improve energy security and lessen trade imbalances. The actual results of the research that examine the link between these factors, however, are occasionally incompatible with one another. Since lack of consensus among the researchers regarding the causal link between the candidates' variables remains major hurdle for policy makers deciding for the best utilization of energy resources while keeping in view higher economic development and environmental protection. The major causes of this contradictory conclusion, according to Kaplan et al. (2011), are the use of various data sets, distinct econometric approaches, and various country-specific factors. According to the findings, there are four main hypotheses that may be used to explain the concluding causal link between ENGY and ECGT (Apergis and Payne, 2010b, 2010a; Bowden and Payne, 2010; Chontanawat et al., 2008; Soytas and Sari, 2009; Tiwari, 2011). First, the growth hypothesis describes a scenario in which ENGY, in addition to capital and labor, plays a crucial part in the process of ECGT. If there is a single route of causality between ECGT and ENGY, the growth hypothesis is upheld. Hence the concluded hypothesis of energy conservation is meant that reduction in ENGY can have a detrimental effect on ECGT. (See the following studies: Apergis & Payne, 2010a, 2010b; Belloumi, 2010; Bowden & Payne, 2010; Chien & Hu, 2008; Chontanawat et al., 2008; Narayan & Smyth, 2009; Soytas & Sari, 2009; Tiwari, 2011 for comparable findings). The second implication of the conservation theory is that the dynamics that drives ENGY is EGT. In the case of one-way causal relationship linking economic growth and energy use, the conservation hypothesis is demonstrated which might not harm economic development. Studies in the existing literature including: (Akinlo, 2008;

Huang et al., 2008; Lise and Van Montfort, 2007; Sadorsky, 2009; Soytas and Sari, 2009) have concluded conservation hypothesis while studying the causal link between ECGT and ENGY. Third, the feedback hypothesis asserts that ECGT and ENGY are mutually related. If there is bi-directional causation link between ENGY and ECGT, the feedback hypothesis would be validated. By validity of this hypothesis, it is intended that changes in ECGT are reflected back in changes in ENGY (Akinlo, 2008; Apergis and Payne, 2010b; Belke et al., 2011; Belke et al., 2012; Eggoh et al., 2011; Eggoh et al., 2021; Fuinhas and Marques, 2012; Tiwari, 2011). Fourthly, the neutrality hypothesis shows that energy consumption has no influence on economic expansion. The neutrality-hypothesis showing no causal link between ECGT and ENGY is supported by the lack of a causal relationship between energy use and economic development. In such case, any measures or policy which are taken for energy saving won't have any impact on economic growth. Neutrality hypothesis have been concluded in the studies of Soytas & Sari (2009) and Bulut & Menegaki (2020).

Overall, the study analyzing the causality link between RNENGY and ECGT gives valuable insights into the potential advantages of RNENGY for ECGT. These benefits can be achieved by addressing the challenges that need to be addressed. There are a variety of study issues that might be to blame for the contradictory findings on the causality relationship that exists between the use of renewable energy and economic growth. Endogeneity, temporal and spatial heterogeneity, methodological issues and model specification lead to inconsistent results and can have a significant impact on the findings of the RNENGY and economic growth nexus. Other issues include a lack of availability of high-quality data at the regional level, issues with methodology that result in ambiguous findings. In light of these research problems, it is essential to give thoughtful consideration to the methodology and design of any subsequent researcher on the renewable & NRNENGY and ECGT nexus. It will enable researchers to produce results that are of a higher quality and more consistent with one another. The fact that earlier researches employed asymptotic approaches for testing putative unit roots and co-integration in small samples is another significant factor that contributes to the lack of consensus about the ENRGY_ECGT nexus. However, there is no assurance that using this method will result in accurate inferences when working with very few samples (Yalta et al., 2010; Abdullah et al., 2017)**.** According to Narayan and Smyth (2009), it is best to use data over a lengthy time range when working with information pertaining to individual nations. Chen et al. (2020) have established the gold standard in this respect; nonetheless, in the majority of situations, data spanning the last 150 years will not be accessible. Long-run time series datasets on energy consumption are unavailable in the majority of nations, including Pakistan. This publication made an attempt to address the two aforementioned gaps in the existing research. First, it utilized a method that is well-suited for investigating the link between energy and growth when the objective is to derive policy implications for a particular nation with a limited accessible time series. Second, it analyzed the causal linkages between ECGT and RNENGY & NRNENGY for a country like Pakistan which confronts severe energy shortfalls due to circular debt for the time period of 1975-2021. This paper is structured in the different sections paper: The next part discusses the novelties in the field of literature. In Section 3, the data, methods, and outcomes will be presented. In the fourth and last section, a conclusion is offered.

2. Literature review

The relationship between ENGY and ECGT has been studied extensively. Decomposing the impacts of RNENGY & NRNENGY on ECGT is the latest tendency in the field of resource and energy economics. Given the importance of these studies to the goals of this research, we provide an in-depth summary of each below. Economists have been drawn to investigate the connections between ENGY and ECGT on the basis of the assumption that ENGY is one of the fundamental markers of economic progress (Halicioglu, 2009). Fuinhas and Marques (2012) investigated the causal link between energy consumption and economic performance of Italy, Spain, Greece, Turkey and Portugal by utilizing time series dataset during 1965-2009. The study employed ARDL bound test as suitable technique due to the permanent shocks and sporadic shocks in the southern European countries. In results, it has been concluded that there exist bi-direction relationship between energy consumption and economic growth in both long and short run. They also claimed that energy conservation policy may lead to reduction in economic growth as marginal production of good is higher than the marginal energy requirement. In the study of Kraft & Kraft (1978), there has been increasing discussion of the link between ENGY and ECGT. In the existing literature, researches that examine the connection between (dis)aggregate ENGY and ECGT can be divided into four distinct categories namely: growth hypothesis, feedback hypothesis, conservative hypothesis and neutral hypothesis. The growth hypothesis was supported by studies by different studies including: Fuinhas & Marques (2012), Yalta (2011), Eggoh et al. (2011), Apergis & Payne (2010b) Apergis and Payne (2015), Chontanawat et al. (2018) and Soyatas et al. (2009). The feedback hypothesis was supported by studies by Belke et al. (2011). The presence of the growth and feedback hypotheses were supported by the findings of Belloumi(2010) and Apergis and Payne (2010b), the existence of the feedback and conversation hypotheses was supported by Belloumi (2010). In addition, the feedback, growth, and conservation hypotheses were supported by the studies of Soytas and Sari (2009) and the existence of the feedback, neutrality and conservation were supported by Akinlo (2009) and Lee (2007). The second kind of research pertains to studies that look at how using renewable energy sources affect GDP expansion. For example, Apergis and Payne (2010b), Belloumi (2010), Soytas et al. (2009) found the accuracy of the feedback hypothesis. Sadorsky (2009) demonstrated that when a country's economy develops, they also increase consumption of renewable energy indicating growth-led hypothesis. Bulut and Menegaki (2020) analyzed the relationship between solar production and economic growth for the 10 resource enriched countries with the highest contribution of solar energy production in the world using panel dataset during 1999-2015. By employing panel cointegraiton and causality method, they found no causal link between solar energy production and economic growth indicating neutrality hypothesis. Whereas Chien & Hu (2008) concluded that increased RNENGY positively contribute to ECGT. Tiwari (2011b) has also conducted research to examine the relative importance of both RNENGY and NRNENGY in economic development for panel dataset of the Eurasian and European countries during 1965-2009. By employing PVAR method, Tiwari (2011b) found that RNENGY causes economic growth positive while NRNENGY has negative impact on economic growth. Huang et al. (2007) investigated the causal link between economic growth and energy consumption in four groups of countries leveled by their income as defined by World Bank for 82 countries. They employed GMM-SYS method for the estimation of Panel VAR model. In results it has been concluded there exist neutrality hypothesis among lower income; growth-led hypothesis among middle income groups; negative impact of economic growth on energy consumption among high income group. Furthermore, the authors have also concluded that higher income group improved their environment through efficient energy use and reduction of CO2 emissions. Apergis & Payne (2015) employed the Toda-Yamamoto approach and incorporated variables of capital and labor force to investigate the correlation between RNENGY and

NRNENGY with ECGT from 1949 to 2006 in United States. The data has supported neutrality hypothesis demonstrating no causal link among each RNENGY and NRNENGY with ECGT. In order to know the causal connection between RNENGY and ECGT, Apergis et al. (2010b) used error correction model for panel data of 19 developed countries during 1984-2007. They investigated the connection between CO2 emissions; nuclear energy consumption, RNENGY and ECGT. They discovered a statistically significant co-integration relationship between these factors. We found that the use of renewable energy sources significantly affects economic growth whereas the use of nuclear energy has a negative effect. Additionally, the results of the short-run causality test showed that there is bidirectional causality between the consumption of renewable and nuclear energy and ECGT, validating the feedback hypothesis; and the results of the long run causality analysis showed that there is unidirectional causality from the RNRNGY to economic growth, thus supporting the growth hypothesis.

Using the Toda-Yamamoto causality model augmented by incorporating investment and labor, Bowden & Payne (2010) investigated the causal links between RNENGY and NRNENGY with ECGT in different sectors of the United States during the periods of 1949-2006. The results concluded no causal link between commercial & industrial RNENGY and ECGT confirming neutrality hypothesis; while the presence of a positive unidirectional causality between residential RNENGY and ECGT is evidence for the growth hypothesis. The results of the causality tests also showed that the feedback hypothesis is supported by the positive and bidirectional causality between NRNENGY in the residential and commercial sectors with real GDP, and that the growth hypothesis is supported by the negative unidirectional causality between industrial NRNENGY and real GDP. Sadorsky (2009) shows evidence of bidirectional causation between RNENGY and ECGT within the context of a panel error correction model for eighteen emerging economies during the period of 1994-2003. Using the PVAR method, Tiwari (2011) compared the effects of increasing use of renewable and nonrenewable energy sources on GDP growth in European and Eurasian nations from 1965 to 2009 and found that the former had a negative influence on GDP growth while the latter had a positive one. soyatas et al. (2009) have extended the work of Ewing et al. (2007) for USA during 2001-2005 and estimated an ARDL model. They concluded that there is a positive correlation between industrial production and use of hydropower, waste, and wind energy, but negative correlation was concluded between employment and these energy sources. Although the long run elasticity estimate for nonrenewable energy consumption is relatively higher, either RNENGY or NRNENGY matters for economic growth.

Hondroyiannis et al. (2002) employed error correction model and Johansen's conintegration method to test the causal link between economic growths, price level and energy consumption for Greece for the periods of 1960-1996. In results, they found no short run relationship among these three variables. However, they concluded that implementation of structural policies aiming for higher economic growth can induce energy conservation with no impediment of economic growth. Apergis and Payne (2010b) analyzed the causal link between RNENGY and NRNENGY with ECGT for 80 countries over the period 1990-2007. Results confirmed the existence of a co-integration relationship among the variables of interest, and causality analysis revealed a short- and long-run bidirectional causal relationship between RNENGY and NRNENGY and ECGT, lending credence to the feedback hypothesis. After reviewing the relevant literature, it is concluded that no research has been conducted to determine the link between ECGT and disaggregated energy use (renewable and non-renewable energy consumption) in Pakistan. Therefore, the following are net contributions of this study that helps to fill gap in the existing literature. In the first place, it examines the nexus between economic expansion and RNENGY and NRNENGY with ECGT in Pakistan. Second, granger causality tests may give an incorrect result for integrated or co-integrated variables if the sample size is too small or there are unexpected asymptotic features (Yalta et al., 2012; Yalta, 2010). Maximum entropy bootstrapping has been employed as a supplementary test with credible outcomes for establishing casual links among the candidates' variables. Both Table 1 and Table 2 provide a concise summary of the research that has been published so far on the topic of the causal relationship between renewable and nonrenewable energy sources and economic development.

Table 1 Non-renewable energy consumption and Economic Growth

Study	Country	Methodology	Results
(Belke et al., 2011)	Twenty five OECD countries	Panel cointegration VEC Model and Granger Causality	Feedback
(Fuinhas and Marques, 2012)	Turkey, Italy, Greece, Spain, Portugal and Greece	ARDL Model	Feedback
(Kaplan et al. 2011)	Turkey	Granger Causality and Johansen and Juselius Cointegration	Feedback
(Eggoh et al., 2011)	21 African countries	Causality and cointegration test for panel dataset	Feedback
(Apergis and Payne, 2010a)	11 Common wealth Stats	fully modified OLS & Panel cointegration	Feedback
(Kaplan et al., 2011)	51 countries	Panel Causality and cointegration test	Feedback & conversation
(Apergis and Payne, 2015)	Nine South American countries	Panel Cointegration and Granger causal test	Growth
(Belloumi, 2010)	Tunisia	Granger causality test	Growth &the feedback
(Bowden and Payne, 2010)	US	Toda–Yamamoto Test	Growth
(Chontanawat et al., 2008)	30 OECD and 78 non-OECD countries	Granger causality	Growth
(Huang et al., 2008)	82 countries	Generalized Method of Moment System	Conservation
(Narayan and Smyth, 2009)	G7	Granger causality and Panel Cointegration test	Growth
(Akinlo, 2008)	11 Sub-Sahara African countries	ARDL Model	Feedback, the conservation & neutrality
(Aqeel and Butt, 2001)	Pakistan	Hsiao's Granger causality	Conservation
(Lise and Van, 2007)	Turkey	Vector error correction model (VEC)	Conservation
(Ewing et al., 2007)	US	Toda–Yamamoto procedure	Neutrality
(Soytas and Sari, 2006)	G-7 countries	Granger Causality and Johansen and Juselius Cointegration	Feedback, conservation, Growth
(Lee and Burnett, 2008)	11 major industrialized countries	Toda–Yamamoto procedure	Feedback conservation, neutrality

(Soytas and Sari, 2009)	G-seven countries and Top 10 emerging markets	Cointegration and vector ECM	Feedback, conservation, Growth

Table 2 Renewable energy-economic growth nexus

Study	Country	Methodology	Result
(Apergis and Payne, 2010a)	six American countries	Panel causality& cointegration tests	Feedback
(Tiwari, 2011)	India	Structural VAR	Growth
(Apergis and Payne, 2015)	US	Multivariate Toda–Yamamoto test	Growth
(Fang, 2011)	China	OLS	Growth
(Bulut and Menegaki, 2020)	27 European countries	Panel Causal Model, Random effect model	Neutrality
(Apergis and Payne, 2010a)	13 countries within Eurasia	Panel causality& cointegration tests	Feedback
(Apergis and Payne, 2010b)	20 OECD countries	Panel causality& cointegration tests	Feedback
(Sadorsky, 2009)	18 emerging countries	fully modified OLS and simple OLS method OLS & dynamic OLS	Conservation
(Chien and Hu, 2008)	45 economies	Data envelopment analysis-DEA	Growth

3. Data specification

The bi-variable causality model of ENGY and ECGT includes different variables including: consumption of renewable energy (RNENGY) and non-renewable energy (NRNENGY) and economic growth rate (ECGT). NRNENGY is measured by the total energy consumption of coal, petroleum and natural gas whereas; renewable energy consumption (RNENGY) is measured by solar, net geo-thermal, wind and biomass energy consumption in millions of kilowatts. Dataset covers the period of 1975-2021 and are extracted from the data source of World Bank known as world development index.

4. Econometric Methodology

Since most issues in economics and the social sciences are non-stationary I(d), dynamic, and adopting, economists run a significant danger of incorrectly rejecting the genuine null hypothesis. To address these issues, standard econometric methodology has always taken into account asymptotic assumption-based tests for stationarity in long-run and short run cointegration. Contrarily, they provide results that change over time and between locations. According to MacKinnon (2002), if the sample size converges to 50, the result of an asymptotic J test will produce more than 80% type-I error at 5% level of significance. Researchers frequently employ a de-trending and differencing strategy to transform a non-stationary series into a stationary series. Decline in OLS efficiency, incorrect specification, and incompatibility with structural changes are few of the problems that arise from using such approaches (Hamilton and Susmel 1994). In addition, the real data structure may be compromised by de-trending or differencing data series, which might lead to a suppression of relevant data (Vinod, 2004; Vinod, 2016; Yalta et al., 2013). Besides these, conventional methodology for causal linkage mainly depends upon the unit root tests from which ADF and PP are widely used. However these tests lose their testing power when more deterministic terms are added into the model. While confronting mixed causality result, Yalta (2011) argued that in future the researchers are supposed to focus on new art of econometric techniques instead of, using the usual conventional methods for different countries and dissimilar time period.

While confronting such issues, Vinod (2004) has developed an alternative technique known as Maximum Entropy Bootstrap Method (Meboost). In this research, we employ the said technique based on simulation to find causal links between RNENGY and NRNENGY with ECGT for Pakistan. The Meboost algorithm constructs time series population ensemble Ω by new computer intensive methodology based on R-Programing for the data series which is even highly dependent non-stationary, existence of jumps and gaps and suffering from regime changes. Maximum entropy bootstrap analysis has the following advantages in comparison of the conventional econometric techniques. First, it ignores unit root, structural changes and cointegration tests which are useless and misinterpreted techniques (Yalta, 2011). Second, it provides vigorous result even with the small sample size. Third, this method avoids transformation of non-stationary data into stationary by both differencing and de-terending the series. For such transformation process may lead to destroy the actual shape of data series that lead to loss true information regarding the variables under consideration (Khan et al., 2019). Fourth, unlikely to Granger causality tests, its confident interval is independent of lag length meaning that it provides consistent result even if number of lags for variable series changes. The following bivariate vector auto regressive model (VAR) models are needed to test causal linkages of RNENGY and NRNENGY with ECGT.

VAR-Model for renewable energy-economic growth nexus

$$\mathrm{RNEGY}_t = \beta_{01} + \sum_{i=1}^{p} \alpha_{1i}\mathrm{RNEGY}_{t-i} + \sum_{i=1}^{p} \beta_{1i} \triangle \mathrm{ECGT}_{t-i} + v_{1i} \quad \text{Equation (1)}$$

$$\mathrm{ECGT}_t = \beta_{02} + \sum_{i=1}^{p} \alpha_{2i}\mathrm{ECGT}_{t-i} + \sum_{i=1}^{p} \beta_{2i}\mathrm{RNEGY}_{t-i} + v_{2i} \quad \text{Equation (2)}$$

VAR-Model non-renewable energy-economic growth nexus

$$\mathrm{NRNEGY}_t = \beta_{11} + \sum_{i=1}^{p} \gamma_{1i}\mathrm{NRNEGY}_{t-i} + \sum_{i=1}^{p} \psi_{1i} \triangle \mathrm{ECGT}_{t-i} + u_{1i} \quad \text{Equation (3)}$$

$$\mathrm{ECGT}_t = \beta_{12} + \sum_{i=1}^{p} \gamma_{2i}\mathrm{ECGT}_{t-i} + \sum_{i=1}^{p} \psi_{2i}\mathrm{NRNEGY}_{t-i} + u_{2i} \quad \text{Equation (4)}$$

Where, u_{ij} and v_{ij} are White Noise error terms of equation of VAR 1 and VAR 2 model respectively. α_i, β_i, γ_i and ψ_i are coefficient of renewable and non-renewable energy and economic growth in VAR model. β_{ij}s are constants terms in the above equations

4.1. High Density Regions

There are many statistical methods to summarize probability distribution with some known probability by the sample space region. Since the resamples of Q=999 are generated via maximum entropy bootstrap analysis for each variables, namely financial development, economic growth; physical capital, and labor force. These resamples are taken to run Q-time regressions for both bivariate and multivariate models. The coefficient estimates, obtained from Q time (999) regressions, are subsequently used to generate confidence interval for each coefficient. In this study, High Density Region (HDR) developed by Hyndman (1996) is used to estimate the confidence interval for the the estimates of parameters. Let's $\hat{\theta}$ is the estimate for the population parameter (θ) then the density function of (θ) becomes $f\left(\hat{\theta}\right)$ become confronting some positive value of Type-I error (α). Then $S(f_\alpha) = (1-\alpha)100\%\ HDR \subset f_\alpha$, which implies that the subset of the largest sample space $(1-\alpha)100\%\ HDR$ is $\hat{\theta}$ (f_α) such as:

$$S(f_\alpha) = \left[\hat{\theta} : f\left(\hat{\theta}\right)\right] \geq f_\alpha$$

Given that 'f_α' is the largest sample space of $\hat{\theta}$ only, if the condition of the following probability statement is satisfied:

$$P\left[\hat{\theta} \in S\left(f_\alpha\right)\right] \geq (1-\alpha)$$

Each point inside the HDR constituency has at least as high a chance as any point lying outside the method, as defined by the HDR (Hyndman 1996). HDR yields trustworthy confidence intervals whether the sample distribution is bimodal or multimodal (Vinod, 2013; Hyndman, 1996 for details). Hyndman (1996) outlined the benefits of living in a densely populated area, as follows: Firstly, it can be summarized comparatively in easy way, by a single number of f_α even in case of complicated high dimensions. Secondly, the HDR volume assumed as small region as possible from total probability region of (1-α). Thirdly if the distribution is normal and either uni-modal or symmetric, HDR consists of with the usual used probabilities region about the means whose area is between $\alpha/2$ and $1-\alpha/2$ quintiles. But in case of multimodal distribution, HDR often provides several disjoint sub-regions through which, it can deliver sufficient information which is incognito by other probability regions (Vinod, 2004; Khan et al., 2019; Ahmed, 2015).

5. Results and discussions

5.1. Model for renewable energy consumption and economic growth nexus

For bivariate model, maximum entropy bootstrap algorithm is used to construct 90% and 95% confidence interval in VAR model-1 and VAR model-2. The confident intervals are given in Table 3 and Table 4 as follows:

Variable	VAR model-1 at 5% level of significance				VAR model-2 at 10% level of significance			
	ECGT		RNEGY		ECGT		RNEGY	
	Lower-Bound	Upper-Bound	Lower-Bound	Upper-Bound	Lower-Bound	Upper-Bound	Lower-Bound	Upper-Bound
Constant	-1.179	1.581	-0.471	1.085	-0.171	0.622	0.965	1.613
ECGT (-1)	**0.951**	**1.007**	**0.757**	**0.951**	**0.433**	**1.145**	**0.932**	**1.197**
RNEGY (-1)	-0.052	0.071	-0.072	0.010	-0.054	0.105	-0.287	0.952
ECGT (-2)					-0.171	0.554	-0.380	-0.100
RNEGY (-2)					-0.136	0.087	-1.007	0.199

Table 3: Meboost Analysis for renewable energy model

Note: a) variables are measured in natural logarithms. b) Variables are in leveled form c) Bold figures means rejection of null hypothesis. d) HDRs are used to construct confidence intervals. VAR model-1 and VAR model-2 are calculated at 5% and 10% level of significance.

On the basis of HDR confidence interval given in Table3: we can make decision about the causal relationship between RNENGY and ECGT. The null hypothesis assuming no causality between the candidate variables can be rejected if zero is outside the confidence interval. Accordingly, the assumed hypothesis that RNENGY does not causes ECGT can be rejected at both 5% and 10% level of significance in VAR model-1 and VAR model-2 analysis. However, the null hypothesis showing ECGT does not affect RNENGY is accepted because zero lies inside the 95% confidence interval of high density region in VAR model-1 as shown by bolded figures. The result supports growth hypothesis where causality runs from RNENGY to ECGT. The VAR model-1 model is extended to VAR model-2 by adding lag 2 for RNENGY and economic growth at 10% level of significance. The results of VAR model-1 is consistent with the result of VAR model-2; indicating that renewable energy causes economic growth. The graphical representation for causal link of RNENGY and ECGT of VAR model-1 and VAR model-2 are presented by Figure 1 and Figure 2 respectively. From the high density regions (HDRs) pictured in Figure 1 and Figure 2 it is clearly shown that zero lies outside the confidence interval of the economic growth coefficients. This supports growth hypothesis. The result is consistent with the finding of Payne (2011) for United States, Fang (2011) for China, Tiwari (2011) for India and Chien &Hu (2007) for the panel data of 45 economies.

5.2. Model for non-renewable energy consumption and economic growth nexus

For bivariate model, maximum entropy bootstrap algorithm is used to construct 90% and 95% confidence interval by the VAR model-1 and VAR model-2 respectively as given in Table 4:

Variable	VAR model-1 at 5% level of significance				VAR model-2 at 10% level of significance			
	ECGT		RNEGY		ECGT		RNEGY	
	Lower-Bound	Upper-Bound	Lower-Bound	Upper-Bound	Lower-Bound	Upper-Bound	Lower-Bound	Upper-Bound
Constant	-0.121	0.592	0.462	1.184	-0.171	0.633	0.645	1.532
ECGT (-1)	-0.064	0.083	-0.083	0.070	-0.065	0.125	-0.254	0.842
RNEGY (-1)	**0.955**	**1.017**	**0.768**	**0.961**	**0.434**	**1.135**	**0.942**	**1.296**
ECGT (-2)	-0.164	0.7832	-1.073	0.370	-0.995	0.725	-0.254	0.745
RNEGY (-2)					-0.146	0.085	-1.017	0.298

Table 4: Meboost Analysis for non-renewable energy model

Note: a) variables are measured in natural logarithms. b) Variables are in leveled form c) bold figures means rejection of null hypothesis. d) HDRs are used to construct confidence intervals.

Table 4: explains the confidence interval of high density regions for the causal link between NRNENGY and ECGT. Accordingly, the null hypothesis assuming no causal link between NRNENGY and ECGT can be rejected if zero lies inside the upper and lower limit confidence interval values. The rejection of null hypothesis is highlighted by bolding HDR values of the confidence interval as shown in Table 4. In results, it is concluded that the null hypothesis of no-causal link from ECGT to NRNENGY is rejected as zero lies outside the HDR regions. Similarly, by these principles, the growth hypothesis showing causality runs from NRNENGY to ECGT is rejected. The empirical results of VAR model-1 model at 5% level of significance is consistent to VAR model-2 at 10% level of significance where both support the existence of conservative hypothesis showing causal link from ECGT to NRNENGY. The HDRs for NRNENGY and ECGT are presented in Figure 3 and 4 respectively. The estimated results have also been found consistent with the finding of Lise and Montfort (2007) for

Turkey, Huang et al. (2008) for the 82 panel countries, Ozturk et al. (2010) for the panel dataset of 51 countries and Aqeel & Butt (2001) for Pakistan.

6. Conclusions and Policy Implications

This article aimed to explore the causality relationship between energy and economic growth in order to derive policy suggestions for Pakistan. In this study, four alternative hypotheses have been investigated to recognize varied outcomes of a causal link between RNENGY and NRNENGY and GRWT. Previous research on Pakistan yielded contradictory results due to a lack of long-term data. The use of asymptotic approaches, which are believed to perform poorly in the case of small samples, as well as issues connected with omitted variables, structural breaks and ad hoc specifications, has concluded inconsistencies in causal link between RNENGY and NRNENGY with ECGT in the existing literature. Maximum Entropy Bootstrap was employed because of its reputation for producing accurate and resilient findings even in tiny samples even in the presence of non-stationarity and potential structural breaks. While employing high density regions (HDRs), for the non-renewable energy model, it has been concluded that there exists causal link from ECGT to NRNENGY supporting conservative hypothesis. In addition, growth led hypothesis has been confirmed in renewable energy model showing causal link from RNENGY to ECGT. It has also been concluded that meboost methodology provides robust results at different lags and level of significances. These conclusions have several policy implications for developing the country's future energy strategy. Firstly, conservation policy for non-renewable energy should be implemented which does not provide any hurdle to the reduction in ECGT in Pakistan Secondly, it is advised that Pakistan's government proactively develop implementation policies for renewable energy through a variety of sources, including the expedited depletion of domestic energy resources, the best possible use of hydropower resources, and the adoption of solar energy in sunny areas.

Conflicts of interest:

The authors have no relevant financial and non-financial interest to disclose

Data availability Statement:

All data are available on the request by the authors.

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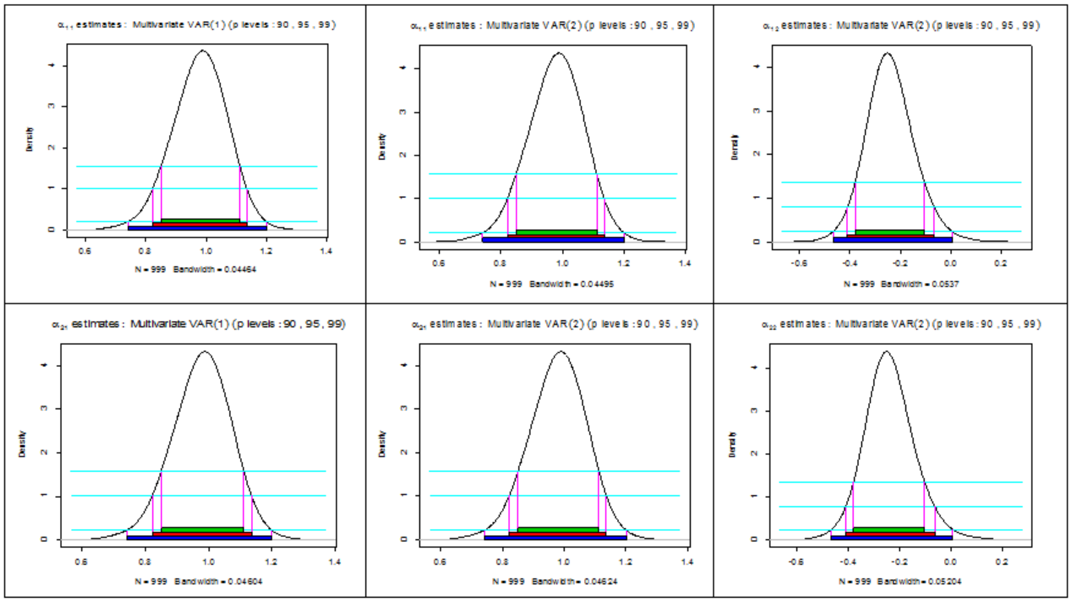
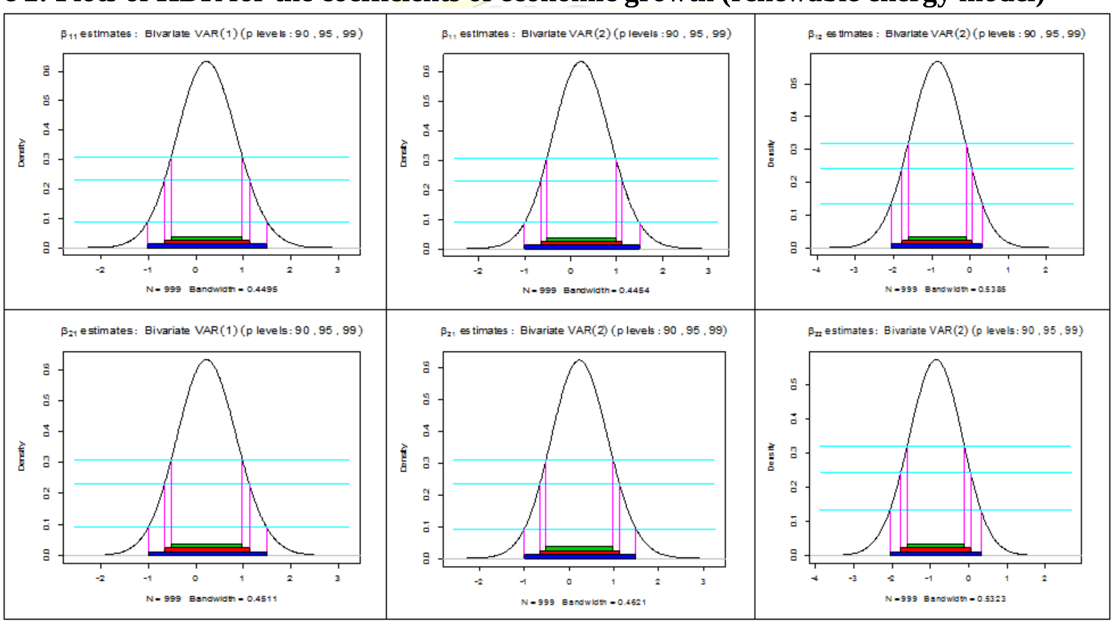
Figure 1: Plots of HDR for the coefficients of renewable energy consumption (renewable energy model)



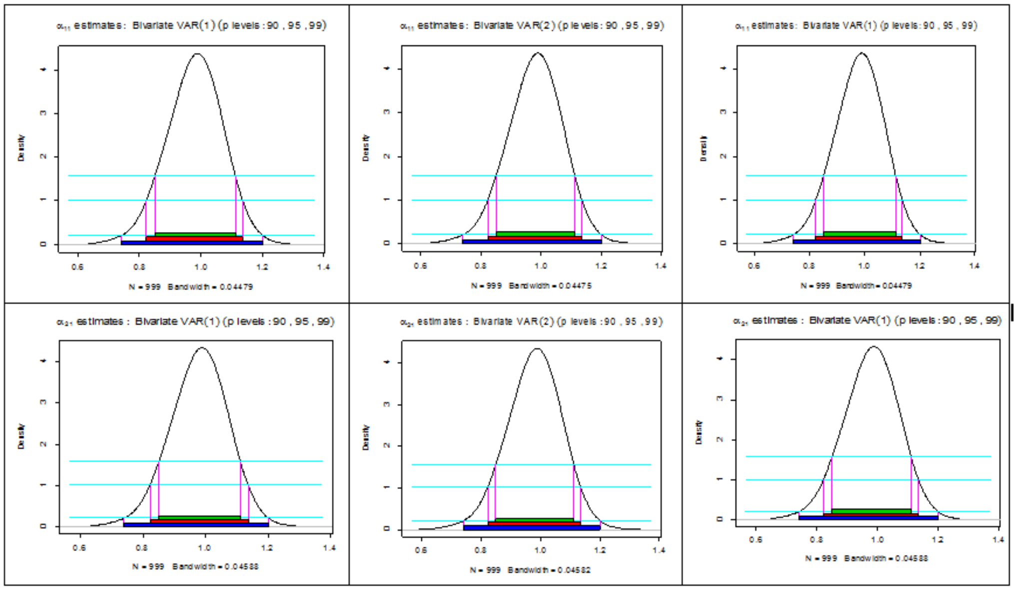
Note: the green, red and blue area show 90%, 95% and 99% HDR confidence interval

Figure 2: Plots of HDR for the coefficients of economic growth (renewable energy model)



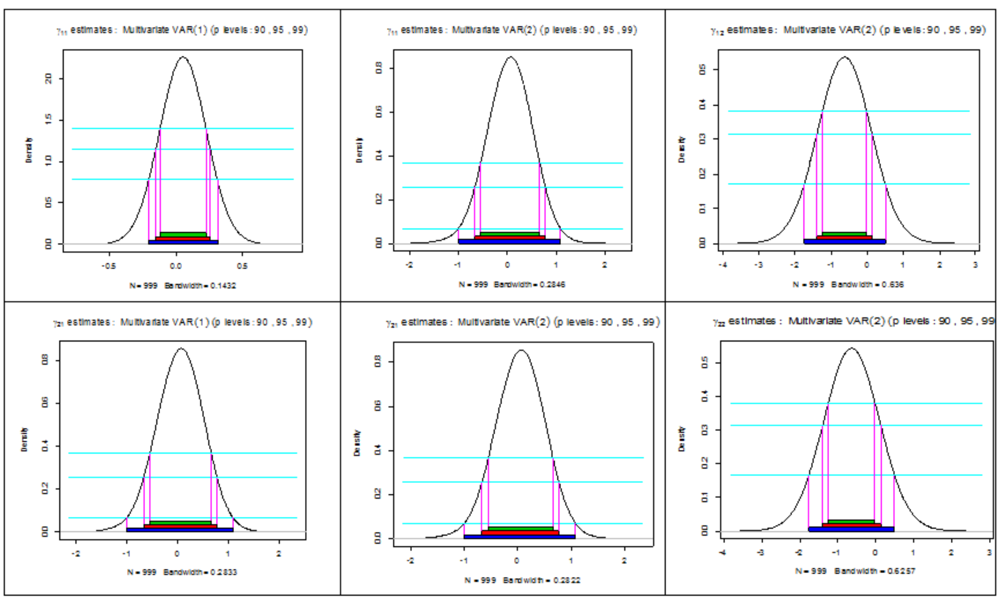
Note: the green, red and blue area show 90%, 95% and 99% HDR confidence interval

Figure 3: Plots of HDR for the coefficients of economic growth (non-renewable energy model)



Note: the green, red and blue area show 90%, 95% and 99% HDR confidence interval

Figure 4: Plots of HDR for the coefficients of non-renewable energy consumption (non-renewable energy model)



Note: the green, red and blue area show 90%, 95% and 99% HDR confidence interval

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