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PARETO LENGTH-BIASED EXPONENTIAL DISTRIBUTION: PROPERTIES AND APPLICATIONS ON THE FLOOD DATA

Shakila Bashir*[1], Bushra Masood[2], Mujahid Rasul[3] and Noor Waseem[4]

[*1]Associate Professor Department of Statistics, Forman Christian College (A Chartered University) Lahore; [2]MPhil Scholar, Forman Christian College (A Chartered University) Lahore; [3]Professor Department of Statistics, Forman Christian College (A Chartered University) Lahore; [4]Kinnaird College for Women Lahore

Corresponding Author: shakilabashir@fccollege.edu.pk

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ABSTRACT

In this article a new continuous probability distribution is proposed by using the T-X family of distributions method. Pareto and length-biased exponential distribution are used as base line distributions. Various statistical including moments, mode, median, quantile function, entropies and several reliability properties such as reliability function, hazard rate function, reversed hazard function, cumulative hazard function, mean residual function, odd function, mills ratio and elasticity along with density and HRF graphs are presented. Parameters estimated by the maximum likelihood estimation and a simulation study is presented for the five estimation methods. Finaly to see the practicality of the model the proposed model is applied on the flood data.

Keyword: T-X family, length-biased, PLED, HRF

INTRODUCTION

In recent years mostly researchers introduced new methods to derive new probability distributions based on techniques where two existing well-known distributions used to derive a new distribution and newly derived distribution is shown more flexible comparatively. The newly derived distributions were based on beta distribution, Lomax distribution, and many more. For example, Akinsete et al. (2008) introduced Beta-Pareto, Barreto-Souza et al. (2009) developed Beta generalized exponential, Cordeiro, and Lemonte (2011) derived Beta-half Cauchy, Al-Kadim and Boshi (2013) exponential Pareto, El-Bassiouny, Abdo, and Shahen (2015) introduced exponential Lomax, Nasiru and Luguterah (2015) developed new Weibull-Pareto distribution. Shabbir et al. (2017) Wibull Lomax, Hamed et al (2018) introduced a new families of generalized Pareto distribution, Waseem and Bashir (2019) proposed T-X Family of Pareto-Exponential distribution Similarly Dar et al. (2020) proposed the weighted gamma pareto distribution with applications on the flood data, Rana and Rahman (2022) investigated the Pareto-Exponential distribution, Ahmad et al. (2021) investigated exponential T-X family of distributions for insurance data, Rana et al. (2022) proposed Pareto-Weibull distribution.

Proposing new discrete and continuous probability distributions are always useful and applicable in various real-life situations where diverse type of data sets can have complex shapes. Researchers and statisticians are always trying to give the best solutions for decision making and estimating the results from various types of data sets and real-life applications. Pareto distribution is skewed to right and it has many applications in economics, geophysical, quality control, actuarial sciences, its is also known as distribution of wealth. Moreover, exponential distribution is also positively skewed distribution and widely used in lifetime analysis.

Dara and Ahmad (2012) introduced length biased exponential distribution (can also called size-biased or moment exponential distribution by assigning the weights to the exponential distribution. Weighted distribution (length is a special case of it) are useful for fitting the models to unknown weight functions when the samples can be taken from the original the developed distributions. Weighted distributions are widely used in the reliability, analysis of family data, meta-analysis and analysis of intervention data, biomedicine, ecology, etc. for fitting the appropriate statistical models.

Alzaatreh et al. (2013) proposed an interesting method to obtain a new probability distribution as given below:

$$F(x) = \int_{a}^{W(G(x))} r(t)dt \tag{1}$$

Where the $W(G(x))$ satisfies the following conditions

$W(G(x)) \in [a, b]$,

$W(G(x))$ is differentiable and monotonically non-decreasing,

$W(G(x)) \to a \ as \ x \to -\infty \ and \ W(G(x)) \to b \ as \ x \to \infty.$

So, considering the eq. (1) as

$$F(x) = \int_{0}^{\frac{1}{1-G(x)}} r(t)dt \tag{2}$$

The probability density function (pdf) of the Pareto distribution is

$$f(x) = \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}, \qquad \beta < x < \infty, \ \ \alpha, \beta > 0. \tag{3}$$

Where α is shape and β is scale parameter. The cumulative distribution function (cdf) of the Pareto distribution is

$$F(x) = 1 - (\beta/x)^{\alpha} \tag{4}$$

The pdf of the size-biased exponential distribution is

$$f(x) = \lambda^2 x e^{-\lambda x}, \qquad x > 0, \ \ \lambda > 0. \tag{5}$$

Where λ is the scale parameter.

In this article two well-known probability distributions are used for developing new probability distributions and the method is considered from the T-X family of distributions. The newly proposed distribution is a generality of the pareto and length biased exponential distributions. Both baseline probability distributions have significant applications in various real-life places. In the section 2 the development of the proposed distribution is provided, section 3, describes various statistical properties of the proposed density, section 4 presents some reliability measures, in section 5 estimation of parameters is presented, section 6 demonstrates the applications and in section 7 conclusion is discussed.

1. Formulation of the Pareto Length-Biased Exponential Distribution

In this section the pareto length-biased distribution is constructed. The pdf and cdf are presented here along with graphical presentation. The cdf of the Pareto length-biased exponential distribution (PLED) is derived by substituting eq. (4) and (5) in eq. (2),

$$F(x) = 1 - e^{-\lambda(x/\beta)^{\alpha}}(1 + \lambda(x/\beta)^{\alpha}) \tag{6}$$

Where, $G(x)$ is the cdf of Pareto distribution given in eq. (4), and $r(t)$ is the pdf of length-biased exponential distribution given in eq. (5).

The pdf of the SEP distribution is

$$f(x) = \frac{\alpha\lambda^2}{\beta}(x/\beta)^{2\alpha-1}e^{-\lambda(x/\beta)^{\alpha}}, \qquad x > 0,\ \alpha, \beta \ \& \ \lambda > 0. \tag{7}$$

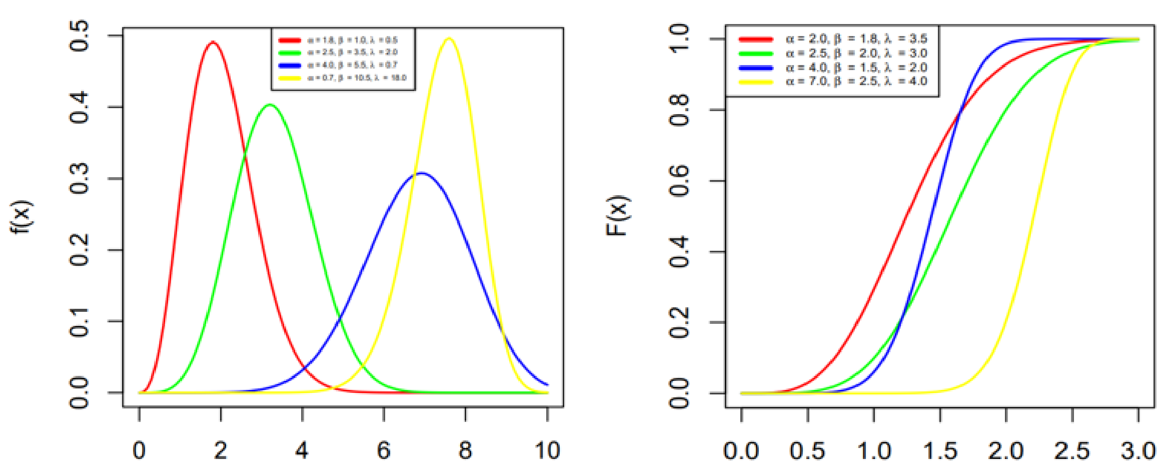

Figure 1. density plot for PLED (left), and CDF plot for PLED (right)

From figure 1, it can be observed that the PLE distribution shows symmetrical, right and left skewness which shows its flexibility on different type of data sets.

2. Statistical Properties of PLED

In this section the moments, quantiles, mode and different entropies for PLED are presented. Numerical description of the measures is also presented. The *rth* order moments of PLED random variable X, in term of Gamma function $\Gamma.$, is given by

$$E(X)^r = \lambda^2\beta^r(1/\lambda)^{\frac{r}{\beta}+2}\Gamma\left(\frac{r}{\beta}+2\right) \tag{8}$$

The mean and variance of PLED are respectively given by

$$Mean = \frac{(\beta+1)\Gamma(1/\beta)}{\beta\lambda^{1/\beta}} \tag{9}$$

$$Var(X) = \frac{1}{\beta^2\lambda^{2/\beta}}[2\beta^2(\beta+2)\Gamma(2/\beta) - (\beta+1)^2\Gamma^2(1/\beta)] \tag{10}$$

The mode of the PLED distribution is

$$mode = \beta\left(\frac{2\alpha-1}{\alpha\lambda}\right)^{1/\alpha} \tag{11}$$

The qth quantile of the PLED distribution is given by

$$x_q = \beta\left[\frac{ln\left(1+\left(x_q/\beta\right)^{\alpha}\right)-ln(1-q)}{\lambda}\right]^{1/\alpha} \tag{12}$$

Median of the PLED is

$$x_{0.5} = \beta\left[\frac{ln(1+(x_{0.5}/\beta)^{\alpha})-ln(0.5)}{\lambda}\right]^{1/\alpha} \tag{13}$$

Table 1 summarizes major properties of PLED varied parameter settings (α, β, λ). Three parameter combinations are investigated, each of which defines a distinct SEP distribution. Set 1 with $(\alpha = 2, \beta = 1.0, \lambda = 1.0)$ has a mean of 1.3293 and a positively skewed distribution with approximately mesokurtic shape. Set 2 $(\alpha = 3, \beta = 2.0, \lambda = 0.5)$ is also positive skew, and platykurtic but not very flat as the value is closer to three (CK = 2.8572), Set 3 $(\alpha = 1, \beta = 0.5, \lambda = 2.0)$ has a lower mean and variance but a greater coefficient of variation (CV = 0.7071) moreover here the shape is positive skewed and leptokurtic.

Table 1: Characteristics of PLED for α, β and λ.

μ_r'	$\alpha=2, \beta=1.0, \lambda=1.0$	$\alpha=3, \beta=2.0, \lambda=0.5$	$\alpha=1, \beta=0.5, \lambda=2.0$
μ_1'	1.3293	3.0002	0.5000
μ_2'	2.0000	9.5535	0.3750
μ_3'	3.3234	32.0000	0.3750
μ_4'	6.0000	112.0083	0.4688
Mean	1.3293	3.0002	0.5000
Var	0.2329	0.5521	0.1250
CV	0.3630	0.2477	0.7071
CS	0.4060	0.0590	1.4142
CK	3.0592	2.8572	6.0000

Entropy functions describe the measures of uncertainties that are useful in reliability and risk analysis.

Renyi Entropy for the PLED

$$\mathrm{RE} = \frac{1}{1-v} log\left[\int_0^\infty \left(f(x)\right)^v dx\right]$$

$$\mathrm{RE} = \frac{1}{1-v} log\left[\int_0^\infty \left[\frac{\alpha\lambda^2}{\beta}\left(\frac{x}{\beta}\right)^{2\alpha-1} e^{-\lambda\left(\frac{x}{\beta}\right)^\alpha}\right]^v dx\right], v>0, v\neq 1$$

Tsallis Entropy for the PLED

$$\mathrm{TE} = \frac{1}{v-1} log\left[1-\int_0^\infty \left(f(x)\right)^v dx\right]$$

$$\mathrm{TE} = \frac{1}{v-1} log\left[1-\int_0^\infty \left[\frac{\alpha\lambda^2}{\beta}\left(\frac{x}{\beta}\right)^{2\alpha-1} e^{-\lambda\left(\frac{x}{\beta}\right)^\alpha}\right]^v dx\right], v>0, v\neq 1$$

Arimoti Entropy for the PLED

$$\mathrm{AE} = \frac{v}{1-v} log\left[\left(\int_0^\infty \left(f(x)\right)^v dx\right)^{1/v} - 1\right]$$

$$\mathrm{AE} = \frac{v}{1-v} log\left[\left\{\int_0^\infty \left[\frac{\alpha\lambda^2}{\beta}\left(\frac{x}{\beta}\right)^{2\alpha-1} e^{-\lambda\left(\frac{x}{\beta}\right)^\alpha}\right]^v dx\right\}^{1/v} - 1\right], v>0, v\neq 1$$

Table 2: Numerical Values of Reyni Entropy, Tsallis Entropy and Arimoto Entropy.

λ	α	β	v=0.5			v=0.8		
			RE	TE	AE	RE	TE	AE
0.8	1.5	0.6	0.8545	-1.0660	1.0660	0.7243	-0.7794	0.7794
		1.5	1.7708	-2.8478	2.8478	1.6406	-1.9417	1.9417
	2.0	0.6	0.4439	-0.4970	0.4970	0.3253	-0.3361	0.3361
		1.5	1.3602	-1.9482	1.9482	1.2416	-1.4093	1.4093
	3.5	0.6	-0.2490	0.2341	-0.2341	-0.3710	0.3575	-0.3575
		1.5	0.6673	-0.7921	0.7921	0.5453	-0.5762	0.5762
	4.5	0.6	-0.5340	0.4686	-0.4686	-0.6613	0.6195	-0.6195
		1.5	0.3823	-0.4213	0.4213	0.2550	-0.2616	0.2616
2.0	1.5	0.6	0.2436	-0.2591	0.2591	0.1134	-0.1147	0.1147
		1.5	1.1599	-1.5719	1.5719	1.0297	-1.1434	1.1434
	2.0	0.6	-0.0142	0.0142	-0.0142	-0.1329	0.1311	-0.1311
		1.5	0.9021	-1.1399	1.1399	0.7834	-0.8481	0.8481
	3.5	0.6	-0.5108	0.4508	-0.4508	-0.6328	0.5944	-0.5944
		1.5	0.4055	-0.4496	0.4496	0.2835	-0.2917	0.2917
	4.5	0.6	-0.7376	0.6169	-0.6169	-0.8650	0.7943	-0.7943
		1.5	0.1787	-0.1869	0.1869	0.0513	-0.0516	0.0516

3. Some Reliability Measures for PLED

In this section several reliability measures such as reliability function, hazard rate function, reversed hazard function, cumulative hazard function, mean residual function, odd function, mills ratio, and elasticity are presented.

The reliability function for PLED is

$$R(x) = e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha}) \quad (14)$$

The hazard rate function (HRF) for PLED is.

$$h(x) = \frac{\alpha\lambda^2(x/\beta)^{2\alpha-1}}{\beta[1+\lambda(x/\beta)^{\alpha}]} \quad (15)$$

The cumulative hazard function for PLED is

$$H(x) = \lambda(x/\beta)^{\alpha} - ln[1+\lambda(x/\beta)^{\alpha}] \quad (16)$$

The reversed hazard rate for PLED is

$$r(x) = \frac{\alpha\lambda^2(x/\beta)^{2\alpha-1}e^{-\lambda(x/\beta)^{\alpha}}}{\beta[1-e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha})]} \quad (17)$$

The mean residual function for the PLED is

$$m(x) = \frac{\beta\left[\Gamma(1/\alpha, \lambda(x/\beta)^{\alpha}) + \Gamma\left(\frac{1}{\alpha}+1, \lambda(x/\beta)^{\alpha}\right)\right]}{\alpha\lambda^{1/\alpha} e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha})} \tag{18}$$

Where $\Gamma(a, x)$ is the upper incomplete gamma function.

The odd function for the PLED is

$$\mathcal{O}(x) = \frac{1-e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha})}{e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha})} \tag{19}$$

Mills ratio for the PLED is

$$M(x) = \frac{\beta e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha})}{\alpha\lambda^{2}(x/\beta)^{2\alpha-1} e^{-\lambda(x/\beta)^{\alpha}}} \tag{20}$$

Elasticity for the PLED is

$$\varepsilon(x) = \frac{\alpha\lambda^{2} x (x/\beta)^{2\alpha-1} e^{-\lambda(x/\beta)^{\alpha}}}{\beta\left[1-e^{-\lambda(x/\beta)^{\alpha}}(1+\lambda(x/\beta)^{\alpha})\right]} \tag{21}$$

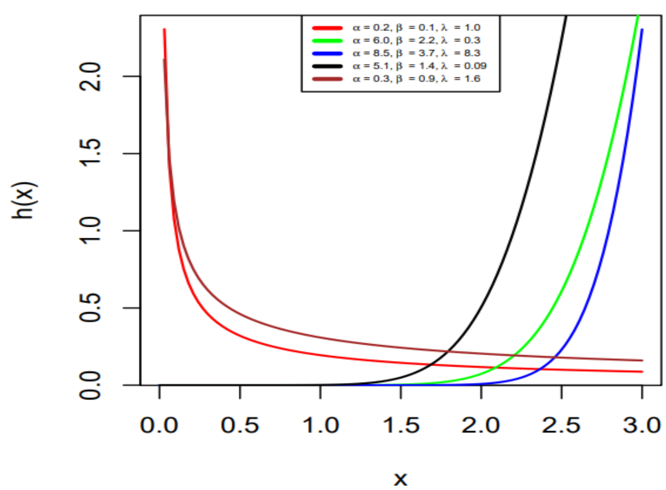

Figure 2. Plot for HRF of PLED

The hazard plot in figure 2, shows that HRF for PLED is showing the decreasing and increasing trend.

4. Random Generation and Parameter Estimation

In this section the estimation of the parameters for the PLED is discussed. The maximum likelihood estimation is proposed to estimate the parameters. Moreover, a Monte Carlo simulation is presented for five estimation methods to estimate the parameters of the PLED. Using the method of inversion random numbers from PLED distribution can be generated as

The *qth* quantile of the SEP distribution is given by

$$x = \beta \left[\frac{ln(1+(x/\beta)^{\alpha}) - ln(1-u)}{\lambda}\right]^{1/\alpha} \tag{22}$$

Where $u \sim U(0,1)$. To generate random number and to apply the newly derived model on real life data sets, we need to estimate the parameters of the PLED distribution. The maximum likelihood estimates (MLE) of the parameters of SEP distribution is by the following: Let $X_1, X_2, ..., X_n$ be a random sample of size n from SEP distribution. Then the log likelihood function $\Im = lnL$ is

$$\Im = nln(\alpha) + 2nln(\lambda) - nln(\beta) - \lambda \sum_{i=1}^{n} (x_i/\beta)^{\alpha} + (2\alpha - 1) \sum_{i=1}^{n} ln(x_i/\beta) \tag{23}$$

Taking derivative w.r.to $\alpha, \beta, and\ \gamma$, we get

$$\frac{\partial \Im}{\partial \alpha} = \frac{n}{\alpha} - \lambda \sum_{i=1}^{n} (x_i/\beta)^{\alpha} ln\big((x_i/\beta)\big) + 2 \sum_{i=1}^{n} ln(x_i/\beta) \tag{24}$$

$$\frac{\partial \Im}{\partial \beta} = \frac{2n}{\lambda} - \sum_{i=1}^{n} (x_i/\beta)^{\alpha} \tag{25}$$

$$\frac{\partial \Im}{\partial \alpha} = \frac{n}{\beta} - \frac{\alpha\lambda}{\beta} \sum_{i=1}^{n} (x_i/\beta)^{\alpha} - (2\alpha - 1)(\sum_{i=1}^{n} ln(x_i) - 1/\beta) \tag{26}$$

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Table 3: Parameter's biases, Average Bias, Mean Standard Error, and Mean Relative Estimates for $\alpha = 1.5$, $\beta = 2.0$ and $\lambda = 0.5$.

Methods		$\alpha = 1.5$				$\beta = 2.0$				$\lambda = 0.5$			
		20	**50**	**100**	**300**	**20**	**50**	**100**	**300**	**20**	**50**	**100**	**300**
MLE	Bias	1.0732	1.0292	1.0160	1.0067	1.8720	1.8577	1.8071	1.7615	1.1939	1.1886	1.1547	1.1276
	Avg. Bias	0.4385	0.4708	0.4840	0.4934	0.2547	0.2184	0.2197	0.2410	0.6940	0.6886	0.6547	0.6276
	MSE	0.2275	0.2361	0.2414	0.2457	0.1062	0.0814	0.0606	0.0602	0.6303	0.5442	0.4525	0.3977
	MRE	0.2923	0.3139	0.3227	0.3289	0.1273	0.1092	0.1099	0.1205	1.3879	1.3771	1.3093	1.2552
AD	Bias	1.5536	1.5291	1.5203	1.5182	2.0538	2.0329	2.0344	2.0437	0.4615	0.4488	0.4474	0.4478
	Avg. Bias	0.2430	0.1516	0.1057	0.0624	0.0665	0.0398	0.0390	0.0452	0.1128	0.0801	0.0639	0.0534
	MSE	0.1020	0.0377	0.0181	0.0062	0.0135	0.0034	0.0027	0.0033	0.0199	0.0097	0.0061	0.0038
	MRE	0.1620	0.1011	0.0705	0.0416	0.0333	0.0199	0.0195	0.0226	0.2256	0.1602	0.1277	0.1069
CVM	Bias	1.6379	1.5612	1.5370	1.5207	2.0560	2.0370	2.0327	2.0413	0.4500	0.4414	0.4409	0.4440
	Avg. Bias	0.3115	0.1820	0.1237	0.0692	0.0630	0.0428	0.0361	0.0421	0.1225	0.0855	0.0690	0.0571
	MSE	0.1862	0.0567	0.0252	0.0077	0.0105	0.0037	0.0023	0.0027	0.0240	0.0110	0.0070	0.0044
	MRE	0.2076	0.1214	0.0825	0.0461	0.0315	0.0214	0.0180	0.0211	0.2450	0.1709	0.1380	0.1142
OLS	Bias	1.5168	1.5162	1.5119	1.5127	2.0452	2.0338	2.0309	2.0408	0.4621	0.4470	0.4445	0.4450
	Avg. Bias	0.2815	0.1735	0.1190	0.0685	0.0538	0.0401	0.0344	0.0418	0.1152	0.0810	0.0661	0.0563
	MSE	0.1384	0.0490	0.0228	0.0075	0.0074	0.0033	0.0021	0.0026	0.0212	0.0099	0.0066	0.0044
	MRE	0.1877	0.1157	0.0794	0.0457	0.0269	0.0201	0.0172	0.0209	0.2304	0.1620	0.1322	0.1126
WLS	Bias	1.5185	1.5401	1.5267	1.5201	2.0668	2.0527	2.0499	2.0496	0.4685	0.4518	0.4489	0.4494
	Avg. Bias	0.2670	0.1610	0.1089	0.0670	0.0792	0.0618	0.0548	0.0526	0.1195	0.0771	0.0619	0.0526
	MSE	0.1172	0.0442	0.0195	0.0071	0.0146	0.0077	0.0062	0.0048	0.0242	0.0094	0.0058	0.0039
	MRE	0.1780	0.1073	0.0726	0.0446	0.0396	0.0309	0.0274	0.0263	0.2390	0.1542	0.1238	0.1052

Table 4: Parameter's biases, Average Bias, Mean Standard Error, and Mean Relative Estimates for $\alpha = 3.5$, $\beta = 1.0$ and $\lambda = 2.5$.

Methods		$\alpha = 3.5$				$\beta = 1.0$				$\lambda = 2.5$			
		20	50	100	300	20	50	100	300	20	50	100	300
MLE	Bias	2.3480	2.2521	2.2222	2.2037	0.8356	0.8288	0.8341	0.8346	2.4650	2.3985	2.4272	2.4289
	Avg. Bias	1.1614	1.2480	1.2778	1.2963	0.1646	0.1714	0.1665	0.1656	0.1297	0.1907	0.1613	0.1572
	MSE	1.5157	1.6222	1.6621	1.6899	0.0309	0.0317	0.0293	0.0285	0.0264	0.0482	0.0418	0.0361
	MRE	0.3318	0.3566	0.3651	0.3704	0.1646	0.1714	0.1665	0.1656	0.0519	0.0763	0.0645	0.0629
AD	Bias	3.3409	3.2857	3.2635	3.2589	1.0600	1.0586	1.0581	1.0587	2.4886	2.4618	2.4514	2.4576
	Avg. Bias	0.5347	0.3669	0.2978	0.2511	0.0787	0.0639	0.0593	0.0587	0.0963	0.0604	0.0600	0.0529
	MSE	0.4408	0.1974	0.1271	0.0818	0.0098	0.0056	0.0044	0.0039	0.0398	0.0103	0.0089	0.0070
	MRE	0.1528	0.1048	0.0851	0.0718	0.0787	0.0639	0.0593	0.0587	0.0385	0.0242	0.0240	0.0212
CVM	Bias	3.4653	3.2998	3.2482	3.2173	1.0576	1.0627	1.0647	1.0640	2.5139	2.4866	2.4761	2.4583
	Avg. Bias	0.6150	0.3984	0.3294	0.2913	0.0865	0.0703	0.0664	0.0640	0.0464	0.0279	0.0270	0.0432
	MSE	0.6429	0.2351	0.1549	0.1081	0.0114	0.0070	0.0056	0.0045	0.0088	0.0023	0.0018	0.0036
	MRE	0.1757	0.1138	0.0941	0.0832	0.0865	0.0703	0.0664	0.0640	0.0186	0.0112	0.0108	0.0173
OLS	Bias	3.2049	3.1978	3.1974	3.2005	1.0872	1.0737	1.0692	1.0655	2.4896	2.4770	2.4700	2.4534
	Avg. Bias	0.6390	0.4373	0.3591	0.3067	0.1044	0.0787	0.0705	0.0655	0.0492	0.0342	0.0323	0.0480
	MSE	0.6216	0.2745	0.1796	0.1179	0.0160	0.0085	0.0062	0.0047	0.0074	0.0030	0.0024	0.0043
	MRE	0.1826	0.1250	0.1026	0.0876	0.1044	0.0787	0.0705	0.0655	0.0197	0.0137	0.0129	0.0192
WLS	Bias	3.2662	3.2683	3.2632	3.2671	1.0747	1.0702	1.0694	1.0636	2.5098	2.5270	2.5325	2.5042
	Avg. Bias	0.5800	0.3810	0.2971	0.2430	0.0924	0.0757	0.0708	0.0639	0.1335	0.0896	0.0880	0.0853
	MSE	0.5117	0.2124	0.1262	0.0770	0.0129	0.0080	0.0066	0.0049	0.0512	0.0198	0.0178	0.0168
	MRE	0.1657	0.1089	0.0849	0.0694	0.0924	0.0757	0.0708	0.0639	0.0534	0.0358	0.0352	0.0341

5. Applications

In this section, the applications of the PLED distribution are illustrated, and its performance is compared to other competing distributions like Weighted Gamma Distribution (WGD), Weighted Exponential Distribution (WED), Generalized Exponential Distribution (GED), Exponential Distribution (ED) and Pareto distribution (PD). The Akaike information criterion (AIC) is one of the model selection procedures used to find the best model. The model with the lowest test statistic is the best for these selection methods. The real-world data set under consideration is flood discharge, as reported by Alzaatreh et al. (2012), and is as follows:

1460, 4050, 3570, 2060, 1300, 1390, 1720, 6280, 1360, 7440, 5320, 1400, 3240, 2710, 4520, 4840, 8320, 13900, 71500, 6250, 2260, 318, 1330, 970, 1920, 15100, 2870, 20600, 3410, 726, 7500, 7170, 2000, 829, 17300, 4740, 13400, 2940, 5660.

Table 5: Model selection criteria

Models	Estimated Parameters			Test statistics	
	α	β	λ	l	AIC
PLED	**0.62970**	**0.78200**	**0.00780**	**379.9510**	**765.902**
WGD	1.79950	0.00030	2.64760	385.9541	777.908
WED	-----	17.4400	0.00020	383.4422	770.8843
GED	-----	-----	0.00023	388.9989	779.9978
GD	-----	0.91960	0.00013	382.9048	769.8097
ED	-----	-----	0.00014	382.9964	767.9929
PD	318	-----	0.41271	392.8100	789.6200

Table 5 indicates that the PLED distribution is the best model, with the highest value of AIC than alternative distributions such as WGD, WED, GED, GD, ED, and a Power Distribution (PD). These findings inform the selection of the best probability distribution to describe the underlying facts without stating precise numerical values.

6. Conclusion

In this research a new continuous probability model is developed by using the technique introduced by Alzaatreh et al (2013). The proposed model is named as pareto length-biased exponential distribution (PLED). Several properties of the PLED are developed and some graphs are also presented. From the numerical values of the measures and graphs it is observed that the PLED distribution is mostly positively skewed distribution, it is showing platykurtic, leptokurtic and mesokurtic shapes. The HRF for the PLED is showing monotonically increasing and decreasing trend which is useful for the life data analysis in the reliability engineering. Different types of entropies are presented with numerical results. Entropies calculate the disorder of the systems and if this order is higher, then the probability is lower. From the numerical values of entropies for the PLED it is observed that the entropy values are lower it means the order is less and so the probabilities will be higher. In other word it is concluded that entropies for PLED are less that's why PLED has less uncertainty in the real-life applications. From the simulation study it can be seen that the mean square error is reducing as the sample size is increasing. Finally, to see the flexibility and practicality of the proposed density, the PLED is modeled on the flood data and the results are compared with some well-known densities. It is observed that the proposed density shows better results as compared to the other densities. Therefore, PLED can be used to estimate and forecast flood related disasters.

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