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GENERALIZED FAMILY OF EXPONENTIAL TYPE ESTIMATORS FOR THE ESTIMATION OF POPULATION COEFFICIENT OF VARIATION

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ABSTRACT
Under simple random sampling, an improved family of estimators is proposed by incorporating auxiliary information to minimize the variation using the known coefficient of variation. The expression for bias and mean square error (MSE) of the generalized class are derived up to first order of approximation. The efficiency conditions of proposed family are also derived with the competitor estimators. The applications of estimator are discussed using simulation study and real-life data sets for the efficiency comparisons of proposed family with some existed estimators. In the light of the results of simulation study and real-life applications it is found that proposed family of estimators have lower mean square errors as compare to the existing once which shows that the proposed class of estimators is more precis. It is also concluded that when correlation between study and auxiliary variables increases, the proposed generalized family of estimators provides more efficient results.

Keywords: Exponential, MSE, PRE, SRS and Coefficient of Variation

INTRODUCTION
There are several measures of dispersion that can be used to measure dispersion in data, out of which coefficient of variation is known to be most efficient, in all the situations where the interest is in relative measure of dispersion, coefficient of variation (CV) is used. Several researchers have used this techniques to improve the efficiency of their estimators like, Shabbir and Gupta (2007) used CV of two auxiliary variables x and z say C_x and C_z, respectively to improve the efficiency of the estimators. Shahzad et al. (2023), Zaman et al. (2022), Bhushan et al. (2021), Archana and Rao (2014, 2011), Garg and Pachori (2020) have also discussed the estimation of coefficient of variation (C_y).

Consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of N units. Let y_i be the observed values of the study variable (y) and (x_i, z_i) be the observed values of the auxiliary variables (x, z) for the ith unit $(i = 1, 2, ..., N)$. We take a sample of size n from a population and using simple random sampling scheme without replacement (SRSWOR). Let $\bar{y}$ and $(\bar{x}, \bar{z})$ be sample means of study variable (y) and auxiliary variables (x, z), respectively corresponds to the population means $\bar{Y}$ and $(\bar{X}, \bar{Z})$. Let,

$s_y^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$, $s_x^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$, $s_z^2 = \frac{\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-1}$ be the sample variances corresponds to the population variances, $S_y^2 = \sum_{i=1}^{N}\frac{(Y_i - \bar{Y})^2}{N-1}$, $S_x^2 = \sum_{i=1}^{N}\frac{(X_i - \bar{X})^2}{N-1}$, $S_z^2 = \frac{\sum_{i=1}^{N}(Z_i - \bar{Z})^2}{N-1}$, respectively.

$C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $C_z = \frac{S_z}{\bar{Z}}$ are the coefficients of variation of y and (x, z), respectively. Let the error terms be defined as,

$$e_0 = \frac{s_y^2 - S_y^2}{S_y^2}, \quad e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}, \quad e_2 = \frac{s_x^2 - S_x^2}{S_x^2},$$

$$e_3 = \frac{\bar{z} - \bar{Z}}{\bar{Z}}, \quad e_4 = \frac{s_z^2 - S_z^2}{S_z^2} \text{ and } e_5 = \frac{\bar{y} - \bar{Y}}{\bar{Y}},$$

Where;

$$E(e_i) = 0, \ i = 0, ..., 5$$

$$E(e_0^2) = \theta(\delta_{400} - 1), \quad E(e_1^2) = \theta C_x^2, \quad E(e_2^2) = \theta(\delta_{040} - 1),$$

$$E(e_3^2) = \theta C_z^2, \quad E(e_4^2) = \theta(\delta_{004} - 1), \quad E(e_5^2) = \theta C_y^2,$$

$$E(e_0e_1) = \theta C_x \delta_{210}, \quad E(e_0e_2) = \theta(\delta_{220} - 1), \ E(e_0e_3) = \theta C_z \delta_{201},$$

$$E(e_0e_4) = \theta(\delta_{202} - 1), \quad E(e_0e_5) = \theta C_y \delta_{300}, \quad E(e_1e_2) = \theta C_x \delta_{030},$$

$$E(e_1e_3) = \theta \rho_{xz} C_x C_z, \quad E(e_1e_4) = \theta C_x \delta_{012}, \ E(e_1e_5) = \theta \rho_{yx} C_y C_x,$$

$$E(e_2e_3) = \theta C_z \delta_{021}, \quad E(e_2e_4) = \theta(\delta_{022} - 1), \ E(e_2e_5) = \theta C_y \delta_{120},$$

$$E(e_3e_4) = \theta C_z \delta_{003}, \quad E(e_3e_5) = \theta \rho_{yz} C_y C_z, \quad E(e_4e_5) = \theta C_y \delta_{102},$$

$$\theta = \frac{1-f}{n} = \frac{n}{N}, \ C_x^* = \frac{s_x}{\bar{x}}, \ C_y^\wedge = \frac{s_y}{\bar{y}},$$

$$C_z^\wedge = \frac{s_z}{\bar{z}}, \ \delta_{rst} = \frac{\mu_{rst}}{\mu_{200}^{r/2}\mu_{020}^{s/2}\mu_{002}^{t/2}}$$

and

$$\mu_{rst} = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})^r(x_i - \bar{X})^s(z_i - \bar{Z})^t}{N-1}.$$

LITERATURE REVIEW

Now, we discuss some existing estimators along with their properties:

(i) The usual sample coefficient of variation is given by:

$$t_1 = \hat{C}_y. \quad (1)$$

The Bias and MSE of t_1 up to first order of approximation are given by:

$$B(t_1) \cong P\theta C_y, \quad (2)$$

$$MSE(t_1) \cong A\theta C_Y^2, \quad (3)$$

Where;

$$P = C_y^2 - \frac{(\delta_{400} - 1)}{8} - \frac{C_y\delta_{300}}{2},$$

$$A = \left(\frac{1}{4}\right)(\delta_{400} - 1) + C_y^2 - C_y\delta_{300}.$$

(ii) Archana and Rao [1] proposed the following ratio estimator:

$$t_2 = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right). \quad (4)$$

The Bias and MSE of t_2 up to first order of approximation are given by

$$B(t_2) \cong C_y\theta P\left(1 - \frac{I}{P}\right), \quad (5)$$

$$MSE(t_2) \cong A\theta C_Y^2\left(1 - \frac{K}{A}\right), \quad (6)$$

Where;

$$I = \frac{1}{2}(C_x\delta_{210}) - C_x^2 - \rho_{yx}C_yC_x,$$

$$K = (C_x\delta_{210}) - C_x^2 - 2\rho_{yx}C_yC_x.$$

(iii) Another Archana and Rao (2011) estimator is $t_3 = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)$. (7)

The Bias and MSE of t_3 up to first order of approximation are given by

$$B(t_3) \cong C_y\theta P\left(1-\frac{L}{P}\right), \quad (8)$$

$$MSE(t_3) \cong A^*\theta C_Y^2\left(1-\frac{M}{A}\right), \quad (9)$$

Where;

$$A^* = \frac{1}{2}(\delta_{220}-1) - C_y\delta_{120},$$
$$M = (\delta_{220}-1) - 2C_y\delta_{120} - (\delta_{040}-1).$$

(iv) The usual ratio estimator is

$$t_4 = \hat{C}_y\left(\frac{C_x}{\hat{C}_x}\right). \quad (10)$$

The Bias and MSE of t_4 up to first order of approximation are given by

$$B(t_4) \cong C_y\theta(P-U-V+W), \quad (11)$$
$$MSE(t_4) \cong C_y^2\theta(A+W-2U), \quad (12)$$

Where;

$$U = \frac{(\delta_{220}-1)}{4} - \frac{C_x\delta_{210}}{2} + \rho_{xy}C_yC_x - \frac{\delta_{120}C_y}{2},$$
$$V = C_x^2 - \frac{(\delta_{040}-1)}{8} - \frac{C_x\delta_{030}}{2},$$
$$W = C_x^2 + \frac{(\delta_{040}-1)}{4} - C_x\delta_{030}.$$

(v) Another form of ratio estimator when using the auxiliary variable (z) is

$$t_5 = \hat{C}_y\left(\frac{C_z}{\hat{C}_z}\right). \quad (13)$$

The Bias and MSE of t_5 up to first order of approximation are given by

$$B(t_5) \cong C_y\theta(P-R-Q+S), \quad (14)$$
$$MSE(t_5) \cong C_y^2\theta(A+S-2R), \quad (15)$$

Where;

$$Q = C_z^2 - \frac{(\delta_{004}-1)}{8} - \frac{C_z\delta_{003}}{2},$$

$$R = \frac{(\delta_{202}-1)}{4} - \frac{C_z\delta_{201}}{2} + \rho_{yz}C_yC_z - \frac{\delta_{102}C_y}{2},$$
$$S = C_z^2 + \frac{(\delta_{004}-1)}{4} - C_z\delta_{003}.$$

(vi) Ratio estimator with two auxiliary variables is

$$t_6 = \hat{C}_y\left(\frac{C_x}{\hat{C}_x}\right)\left(\frac{C_z}{\hat{C}_z}\right). \quad (16)$$

The Bias and MSE of t_6 up to first order of approximation are given by

$$B(t_6) \cong C_y\theta(P-U+W-V-R+O-Q), \quad (17)$$

$$MSE(t_6) \cong C_y^2\theta(A+W+S+2(O-U-R)), \quad (18)$$

Where

$$O = \frac{(\delta_{022}-1)}{4} - \frac{C_z\delta_{021}}{2} + \rho_{xz}C_xC_z - \frac{\delta_{012}C_x}{2}.$$

(vii) Ratio estimator with two auxiliary variables is

$$t_7 = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right). \quad (19)$$

The Bias and MSE of t_7 up to first order of approximation are given by

$$B(t_7) \cong C_y\theta\Big(P + C_y^2 + C_z^2 - \frac{\delta_{210}C_x}{2} - \frac{\delta_{201}C_z}{2} + \rho_{yz}C_yC_z + \rho_{yx}C_yC_x + \rho_{xz}C_xC_z\Big), \quad (20)$$

$$MSE(t_7) \cong C_y^2\theta(A + C_x^2 + C_z^2 + 2(\rho_{yz}C_yC_z + \rho_{yx}C_yC_x + \rho_{xz}C_xC_z) - \delta_{210}C_x - \delta_{201}C_z). \quad (21)$$

(viii) Another ratio type estimator with two auxiliary information is

$$t_8 = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)\left(\frac{S_z^2}{s_z^2}\right). \quad (22)$$

The Bias and MSE of t_8 up to first order of approximation are given by

$$B(t_8) \cong \theta C_y(P + \delta_{102}C_y + (\delta_{022} - 1) + (\delta_{040} - 1) + (\delta_{004} - 1) - \frac{(\delta_{202} - 1)}{2} - \frac{(\delta_{220} - 1)}{2} + \delta_{120}C_y), \quad (23)$$

$$MSE(t_8) \cong C_y^2\theta(A + (\delta_{004} - 1) + (\delta_{040} - 1) - (\delta_{220} - 1) - (\delta_{202} - 1) + 2(\delta_{102}C_y + \delta_{120}C_y + (\delta_{022} - 1)). \quad (24)$$

PROPOSED ESTIMATOR

Motivated by Muneer et al. (2018), a new exponential type of ratio estimator of finite population coefficient of variation (CV) using the study variable (y) and two auxiliary variables (x, z) is proposed. Some members of the class of estimators from proposed estimator are also generated,

$$t_{prop} = \hat{C}_y\left[\exp\left(\frac{G_1 - D_1}{G_1 + D_1}\right)\exp\left(\frac{G_2 - D_2}{G_2 + D_2}\right)\right], \quad (25)$$

Where.

$$G_1 = (A_1 + C_1)C_x + fB_1\hat{C}_x,$$
$$G_2 = (A_2 + C_2)C_z + fB_2\hat{C}_z,$$
$$D_1 = (A_1 + fB_1)C_x + C_1\hat{C}_x,$$
$$D_2 = (A_2 + fB_2)C_z + C_2\hat{C}_z,$$
$$A_i = (d_i - 1)(d_i - 2);\ B_i = (d_i - 1)(d_i - 4);$$
$$C_i = (d_i - 2)(d_i - 3)(d_i - 4),\ (i = 1, 2, 3, 4).$$

Here, d_i are constants. Now solving (25) in terms of e's, we have

$$t_{prop} = C_y\Bigg[1 + T_2\left(\frac{e_4}{2} - \frac{e_4^2}{8} - e_3 - \frac{e_3e_4}{2} + e_3^2 - \frac{e_5e_4}{2} + e_5e_3 + \frac{e_0e_4}{4} - \frac{e_0e_3}{2}\right) + T_1\left(\frac{e_2}{2} - \frac{e_2^2}{8} - e_1 - \frac{e_1e_2}{2} + e_1^2 - \frac{e_5e_2}{2} + e_5e_1 + \frac{e_0e_2}{4} - \frac{e_0e_1}{2}\right) - J_2T_2\left(e_3^2 + \frac{e_4^2}{4} - e_4e_3\right) - J_1T_1\left(e_1^2 + \frac{e_2^2}{4} - e_2e_1\right) + \frac{T_2^2}{2}\left(e_3^2 + \frac{e_4^2}{4} - e_4e_3\right) + \frac{T_1^2}{2}\left(e_1^2 + \frac{e_2^2}{4} - e_2e_1\right) + T_1T_2\left(\frac{e_2e_4}{4} - \frac{e_2e_3}{2} - \frac{e_1e_4}{2} + e_1e_3\right) - e_5 + e_5^2 + \frac{e_0}{2} - \frac{e_0^2}{8} - \frac{e_5e_0}{2}\Bigg], \quad (26)$$

where

$$T_1 = \frac{fB_1 - C_1}{2(A_1 + C_1 + fB_1)},$$
$$J_1 = \frac{fB_1 + C_1}{2(A_1 + C_1 + fB_1)},$$
$$T_2 = \frac{fB_2 - C_2}{2(A_2 + C_2 + fB_2)},$$
$$J_2 = \frac{fB_2 + C_2}{2(A_2 + C_2 + fB_2)},$$

$$t_{prop} - C_y = C_y\Bigg[T_2\left(\frac{e_4}{2} - \frac{e_4^2}{8} - e_3 - \frac{e_3e_4}{2} + e_3^2 - \frac{e_5e_4}{2} + e_5e_3 + \frac{e_0e_4}{4} - \frac{e_0e_3}{2}\right) + T_1\left(\frac{e_2}{2} - \frac{e_2^2}{8} - e_1 - \frac{e_1e_2}{2} + e_1^2 - \frac{e_5e_2}{2} + e_5e_1 + \frac{e_0e_2}{4} - \frac{e_0e_1}{2}\right) - J_2T_2\left(e_3^2 + \frac{e_4^2}{4} - e_4e_3\right) - J_1T_1\left(e_1^2 + \frac{e_2^2}{4} - e_2e_1\right) + \frac{T_1^2}{2}\left(e_1^2 + \frac{e_2^2}{4} - e_2e_1\right) + \frac{T_2^2}{2}\left(e_3^2 + \frac{e_4^2}{4} - e_4e_3\right) + T_1T_2\left(\frac{e_2e_4}{4} - \frac{e_2e_3}{2} - \frac{e_1e_4}{2} + e_1e_3\right) - e_5 + e_5^2 + \frac{e_0}{2} - \frac{e_0^2}{8} - \frac{e_5e_0}{2}\Bigg]. \quad (27)$$

By Applying Expectation on equation (27) we will get;

$$B(t_{prop}) \cong C_y\theta[P + T_2(Q + R) + T_1(U + V) - T_1J_1(W) - T_2J_2(S) + \frac{T_1^2}{2}(W) + \frac{T_2^2}{2}(S) + T_1T_2(O). \quad (28)$$

Taking square of (27) and applying expectations, up to first order of approximation, we get;

$$MSE(t_{prop}) = C_y^2\theta[T_2^2S + T_1^2W + 2T_1T_2O + 2T_1U + 2T_2R + A]. \quad (29)$$

Differentiate (29) w.r.t T_1 and T_2, respectively; and equate to zero, we get;

$$\frac{\partial MSE(t_{prop})}{\partial T_1} = C_y^2\theta[2T_1W + 2T_2O + 2U],$$

$$\frac{\partial MSE(t_{prop})}{\partial T_2} = C_y^2\theta[2T_2S + 2T_1O + 2R],$$

$$T_1(opt) = \frac{RO - SU}{WS - O^2},$$

$$T_2(opt) = \frac{UO - WR}{WS - O^2},$$

where

$$U = \frac{(\delta_{220} - 1)}{4} - \frac{C_x\delta_{210}}{2} + \rho_{xy}C_yC_x - \frac{\delta_{120}C_y}{2},$$

$$W = C_x^2 + \frac{(\delta_{040} - 1)}{4} - C_x\delta_{030},$$

$$R = \frac{(\delta_{202} - 1)}{4} - \frac{C_z\delta_{201}}{2} + \rho_{yz}C_yC_z - \frac{\delta_{102}C_y}{2},$$

$$S = C_z^2 + \frac{(\delta_{004} - 1)}{4} - C_z\delta_{003},$$

$$O = \frac{(\delta_{022} - 1)}{4} - \frac{C_z\delta_{021}}{2} + \rho_{xz}C_xC_z - \frac{\delta_{012}C_x}{2}.$$

Substituting the optimum value of T_1 and T_2 in (29), the minimum MSE of t_{prop}, is as;

$$MSE_{\min(t_{prop})} \cong C_y^2\theta\left[A - \frac{SU^2 + WR^2 - 2RUO}{WS - O^2}\right]. \quad (30)$$

Now by putting different values of d in equation (25) some members of the proposed class of estimators can be generated.

Table 1. Class of estimators for different values of d_1 and d_2

d_2	Estimator	Bias	MSE
1	$t_{prop1}=\hat{C}_y\exp\left(\frac{C_x-\hat{C}_x}{C_x+\hat{C}_x}\right)\exp\left(\frac{C_z-\hat{C}_z}{C_z+\hat{C}_z}\right)$	$\theta C_y\left(P-\frac{1}{2}(Q+R)-\frac{1}{2}(U+V)+\frac{3}{8}(W+S)+\frac{1}{4}O\right)$	$\theta C_y^2\left(\frac{1}{4}(W+S)-U-R+\frac{1}{2}O+A\right)$
2	$t_{prop2}=\hat{C}_y\exp\left(\frac{C_x-\hat{C}_x}{C_x+\hat{C}_x}\right)\exp\left(\frac{\hat{C}_z-C_z}{\hat{C}_z+C_z}\right)$	$\theta C_y\left(P+\frac{1}{2}(Q+R)-\frac{1}{2}(U+V)+\frac{3}{8}W-\frac{1}{8}S-\frac{1}{4}O\right)$	$\theta C_y^2\left(\frac{1}{4}(W+S)-U+R-\frac{1}{2}O+A\right)$
3	$t_{prop3}=\hat{C}_y\exp\left(\frac{C_x-\hat{C}_x}{C_x+\hat{C}_x}\right)\exp\left(\frac{n(C_z-\hat{C}_z)}{2NC_z-n(\hat{C}_z+C_z)}\right)$	$\theta C_y\left(P-\frac{f}{2(1-f)}(Q+R)-\frac{1}{2}(U+V)+\frac{3}{8}W-\left(\frac{f}{2(1-f)}\right)^2S\right.$ $\left.+\frac{s}{2}\left(\frac{f}{2(1-f)}\right)^2+\frac{f}{4(1-f)}O\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2S+\frac{1}{4}W-U-\frac{f}{(1-f)}R+\frac{f}{2(1-f)}O+A\right)$
4	$t_{prop4}=\hat{C}_y\exp\left(\frac{C_x-\hat{C}_x}{C_x+\hat{C}_x}\right)$	$\theta C_y\left(P-\frac{1}{2}(U+V)+\frac{3}{8}W\right)$	$\theta C_y^2\left(\frac{1}{4}W-U+A\right)$
1	$t_{prop5}=\hat{C}_y\exp\left(\frac{\hat{C}_x-C_x}{\hat{C}_x+C_x}\right)\exp\left(\frac{C_z-\hat{C}_z}{C_z+\hat{C}_z}\right)$	$\theta C_y\left(P-\frac{1}{2}(Q+R)+\frac{1}{2}(U+V)-\frac{1}{8}W+\frac{3}{8}S-\frac{1}{4}O\right)$	$\theta C_y^2\left(\frac{1}{4}(W+S)+U-R-\frac{1}{2}O+A\right)$
2	$t_{prop6}=\hat{C}_y\exp\left(\frac{\hat{C}_x-C_x}{\hat{C}_x+C_x}\right)\exp\left(\frac{\hat{C}_z-C_z}{\hat{C}_z+C_z}\right)$	$\theta C_y\left(P+\frac{1}{2}(Q+R)+\frac{1}{2}(U+V)-\frac{1}{8}W-\frac{1}{8}S+\frac{1}{4}O\right)$	$\theta C_y^2\left(\frac{1}{4}(W+S)+U+R+\frac{1}{2}O+A\right)$
3	$t_{prop7}=\hat{C}_y\exp\left(\frac{\hat{C}_x-C_x}{\hat{C}_x+C_x}\right)\exp\left(\frac{n(C_z-\hat{C}_z)}{2NC_z-n(\hat{C}_z+C_z)}\right)$	$\theta C_y\left(P-\frac{f}{2(1-f)}(Q+R)+\frac{1}{2}(U+V)-\frac{1}{8}W+\left(\frac{f}{2(1-f)}\right)^2S\right.$ $\left.+\frac{s}{2}\left(\frac{f}{2(1-f)}\right)^2-\frac{f}{4(1-f)}O\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2S+\frac{1}{4}W+U-\frac{f}{(1-f)}R-\frac{f}{2(1-f)}O+A\right)$
4	$t_{prop8}=\hat{C}_y\exp\left(\frac{\hat{C}_x-C_x}{\hat{C}_x+C_x}\right)$	$\theta C_y\left(P+\frac{1}{2}(U+V)-\frac{1}{8}W\right)$	$\hat{C}_y\exp\left(\frac{n(C_x-\hat{C}_x)}{2NC_x-n(\hat{C}_x+C_x)}\right)\exp\left(\frac{C_z-\hat{C}_z}{C_z+\hat{C}_z}\right)$
1	$t_{prop9}=\hat{C}_y\exp\left(\frac{n(C_x-\hat{C}_x)}{2NC_x-n(\hat{C}_x+C_x)}\right)\exp\left(\frac{C_z-\hat{C}_z}{C_z+\hat{C}_z}\right)$	$\theta C_y\left(P-\frac{f}{2(1-f)}(U+V)-\frac{1}{2}(Q+R)+\frac{3}{8}S-\left(\frac{f}{2(1-f)}\right)^2W\right.$ $\left.+\frac{w}{2}\left(\frac{f}{2(1-f)}\right)^2+\frac{f}{4(1-f)}O\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2W+\frac{1}{4}S-R-\frac{f}{(1-f)}U+\frac{f}{2(1-f)}O+A\right)$
2	$t_{prop10}=\hat{C}_y\exp\left(\frac{n(C_x-\hat{C}_x)}{2NC_x-n(\hat{C}_x+C_x)}\right)\exp\left(\frac{\hat{C}_z-C_z}{\hat{C}_z+C_z}\right)$	$C_y\left(P-\frac{f}{2(1-f)}(U+V)+\frac{1}{2}(Q+R)-\frac{1}{8}S+\left(\frac{f}{2(1-f)}\right)^2W\right.$ $\left.+\frac{w}{2}\left(\frac{f}{2(1-f)}\right)^2-\frac{f}{4(1-f)}O\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2W+\frac{1}{4}S+R-\frac{f}{(1-f)}U-\frac{f}{2(1-f)}O+A\right)$
3	$t_{prop11}=\hat{C}_y\exp\left(\frac{n(C_x-\hat{C}_x)}{2NC_x-n(\hat{C}_x+C_x)}\right)$ $\times\exp\left(\frac{n(C_z-\hat{C}_z)}{2NC_z-n(\hat{C}_z+C_z)}\right)$	$\theta C_y\left(P-\frac{f}{2(1-f)}(Q+R)-\frac{f}{2(1-f)}(U+V)+\left(\frac{f}{2(1-f)}\right)^2S+\left(\frac{f}{2(1-f)}\right)^2W\right.$ $\left.+\frac{s}{2}\left(\frac{f}{2(1-f)}\right)^2+\frac{w}{2}\left(\frac{f}{2(1-f)}\right)^2+\left(\frac{f}{2(1-f)}\right)^2O\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2S+\left(\frac{f}{2(1-f)}\right)^2W-\frac{f}{(1-f)}R\right.$ $\left.-\frac{f}{(1-f)}U+\frac{o}{2}\left(\frac{f}{(1-f)}\right)^2+A\right)$
4	$t_{prop12}=\hat{C}_y\exp\left(\frac{n(C_x-\hat{C}_x)}{2NC_x-n(\hat{C}_x+C_x)}\right)$	$\theta C_y\left(P-\frac{f}{2(1-f)}(U+V)+\left(\frac{f}{2(1-f)}\right)^2W+\frac{w}{2}\left(\frac{f}{2(1-f)}\right)^2\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2W-\frac{f}{(1-f)}U+A\right)$
1	$t_{prop13}=\hat{C}_y\exp\left(\frac{C_z-\hat{C}_z}{C_z+\hat{C}_z}\right)$	$\theta C_y\left(P-\frac{1}{2}(Q+R)+\frac{3}{8}S\right)$	$\theta C_y^2\left(\frac{1}{4}S-R+A\right)$
2	$t_{prop14}=\hat{C}_y\exp\left(\frac{\hat{C}_z-C_z}{\hat{C}_z+C_z}\right)$	$\theta C_y\left(P+\frac{1}{2}(Q+R)-\frac{1}{8}S\right)$	$\theta C_y^2\left(\frac{1}{4}S+R+A\right)$
3	$t_{prop15}=\hat{C}_y\exp\left(\frac{n(C_z-\hat{C}_z)}{2NC_z-n(\hat{C}_z+C_z)}\right)$	$\theta C_y\left(P-\frac{f}{2(1-f)}(Q+R)+\left(\frac{f}{2(1-f)}\right)^2S+\frac{s}{2}\left(\frac{f}{2(1-f)}\right)^2\right)$	$\theta C_y^2\left(\left(\frac{f}{2(1-f)}\right)^2S-\frac{f}{(1-f)}R+A\right)$
4	$t_{prop16}=\hat{C}_y$	$\theta C_y P$	θC_y^2

3. Theoretical Comparison

We compare the proposed estimator with other estimators in terms of MSE:

(i) $MSE(t_{prop}) < MSE(t_1)$ if

$MSE(t_1) - MSE(t_{prop}) > 0$ or

$$\left(\frac{SU^2 + WR^2 - 2RUO}{WS - O^2}\right) > 0. \quad (31)$$

(ii) $MSE(t_{prop}) < MSE(t_2)$ if

$MSE(t_2) - MSE(t_{prop}) > 0$ or

$$\left(\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} - K\right) > 0. \quad (32)$$

(iii) $MSE(t_{prop}) < MSE(t_3)$ if

$MSE(t_3) - MSE(t_{prop}) > 0$ or

$$\left(\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} - M\right) > 0. \quad (33)$$

(iv) $MSE(t_{prop}) < MSE(t_4)$ if

$MSE(t_4) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} - 2U + W > 0. \quad (34)$$

(v) $MSE(t_{prop}) < MSE(t_5)$ if

$MSE(t_5) - MSE(t) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} - 2R + S > 0. \quad (35)$$

(vi) $MSE(t_{prop}) < MSE(t_6)$ if

$MSE(t_6) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + (W + S) + 2(O - U - R) > 0. \quad (36)$$

(vii) $MSE(t_{prop}) < MSE(t_7)$ if

$MSE(t_7) - MSE(t_{prop}) > 0$ o

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + (C_x^2 + C_z^2 + 2(\rho_{yz}C_yC_z + \rho_{yx}C_yC_x + \rho_{xz}C_xC_z - \delta_{210}C_x - \delta_{201}C_z) > 0.$$

(37)

(viii) $MSE(t_{prop}) < MSE(t_8)$ if

$MSE(t_8) - MSE(t_{prop}) > 0$ o

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2(31)} + \{(\delta_{004} - 1) + (\delta_{040} - 1) - (\delta_{220} - 1) - (\delta_{202} - 1) + 2(\delta_{020}C_y + \delta_{102}C_y + (\delta_{022} - 1))\} > 0.$$

(38)

(ix) $MSE(t_{prop}) < MSE(t_{prop1})$ if

$MSE(t_{prop1}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}(W + S) - U - R + \frac{1}{2}O > 0.$$

(39)

(x) $MSE(t_{prop}) < MSE(t_{prop2})$ if

$MSE(t_{prop2}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}(W + S) - U + R - \frac{1}{2}O > 0.$$

(40)

(xi) $MSE(t_{prop}) < MSE(t_{prop3})$ if

$MSE(t_{prop3}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 S + \frac{1}{4}W - U - \frac{f}{(1-f)}R + \frac{f}{2(1-f)}O > 0.$$

(41)

(xii) $MSE(t_{prop}) < MSE(t_{prop4})$ if

$MSE(t_{prop4}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}W - U > 0.$$

(42)

(xiii) $MSE(t_{prop}) < MSE(t_{prop5})$ if

$MSE(t_{prop5}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}(W + S) - U - R - \frac{1}{2}O > 0.$$

(43)

(xiv) $MSE(t_{prop}) < MSE(t_{prop6})$ if

$MSE(t_{prop6}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}(W + S) + U + R + \frac{1}{2}O > 0.$$

(44)

(xv) $MSE(t_{prop}) < MSE(t_{prop7})$ if

$MSE(t_{prop7}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 S + \frac{1}{4}W + U - \frac{f}{(1-f)}R - \frac{f}{2(1-f)}O > 0. \quad (45)$$

(xvi) $MSE(t_{prop}) < MSE(t_{prop8})$ if

$MSE(t_{prop8}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}W + U > 0. \quad (46)$$

(xvii) $MSE(t_{prop}) < MSE(t_{prop9})$ if

$MSE(t_{prop9}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 W + \frac{1}{4}S - R - \frac{f}{(1-f)}U + \frac{f}{2(1-f)}O > 0. \quad (47)$$

(xviii) $MSE(t_{prop}) < MSE(t_{prop10})$ if

$MSE(t_{prop10}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 W + \frac{1}{4}S + R - \frac{f}{(1-f)}U - \frac{f}{2(1-f)}O > 0. \quad (48)$$

(xix) $MSE(t_{prop}) < MSE(t_{prop11})$ if

$MSE(t_{prop11}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 S + \left(\frac{f}{2(1-f)}\right)^2 W - \frac{f}{(1-f)}R - \frac{f}{(1-f)}U + \frac{O}{2}\left(\frac{f}{(1-f)}\right)^2 > 0.$$

(49)

(xx) $MSE(t_{prop}) < MSE(t_{prop12})$ if

$MSE(t_{prop12}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 W - \frac{f}{(1-f)}U > 0.$$

(50)

(xxi) $MSE(t_{prop}) < MSE(t_{prop13})$ if

$MSE(t_{prop13}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}S - R > 0.$$

(51)

(xxii) $MSE(t_{prop}) < MSE(t_{prop14})$ if

$MSE(t_{prop14}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \frac{1}{4}S + R > 0.$$

(52)

(xxiii) $MSE(t_{prop}) < MSE(t_{prop15})$ if

$MSE(t_{prop15}) - MSE(t_{prop}) > 0$ or

$$\frac{SU^2 + WR^2 - 2RUO}{WS - O^2} + \left(\frac{f}{2(1-f)}\right)^2 S - \frac{f}{(1-f)}R > 0.$$

(53)

4. Numerical Illustration

Data set 1 (Source: Wheat Production (2014-2017))**.** We took a dataset of district-wise yearly wheat production in the districts of Punjab for three years from Statistical Bureau of Pakistan. There are total 37 districts in the Punjab province, out of which 8 have been picked as a sample for estimation. For this study, the current year will be considered as a study variable and past years are considered as auxiliary variables. This data set will help find MSE and PRE of different estimators for comparison.

The variables are

$y =$ Wheat production of year (2016-17).

$x =$ Wheat production of year (2015-16).

$z =$ Wheat production of year (2014-15).

The data statistics are

$\delta_{400} = 2.4428,\ \delta_{300} = 0.0817,\ \bar{X} = 527.7480,\ \bar{Y} = 553.1440,$

$\delta_{210} = 0.1671,\ \delta_{102} = 0.2825,\ \bar{Z} = 521.1330,\ N = 37,$

$\delta_{202} = 2.3416,\ \delta_{040} = 2.4699,\ S_z = 279.0132,\ S_x = 268.4335,$

$\delta_{201} = 0.1803,\ \delta_{003} = 0.3855,\ C_z = 0.5354,\ C_y = 0.5161,$

$\delta_{220} = 2.3791,\ \delta_{012} = 0.3643,\ \rho_{xz} = 0.9900,\ \rho_{yz} = 0.9732,$

$\delta_{120} = 0.2490,\ \delta_{030} = 0.3320,\ \rho_{xy} = 0.9809,\ S_y = 285.4801,$

$\delta_{004} = 2.4217,\ \delta_{022} = 2.4226,\ \delta_{021} = 0.3457,\ C_x = 0.5086.$

Data set 2 (Source: Sarndal et al (1992))**.**

The MU284 population (the population consisting of the 284 municipalities of Sweden) let

$y = 1985$ population (in thousands).

$x =$ Number of social-democratic seats in municipal council.

$z =$ Total number of seats in municipal.

The data statistics are

$\delta_{400} = 88.92,\ \delta_{300} = 8.22,\ \bar{X} = 22.19,\ \bar{Y} = 29.36,$

$\delta_{210} = 2.000,\ \delta_{102} = 1.715,\ \bar{Z} = 47.53,\ N = 284,$

$\delta_{202} = 14.941,\ \delta_{040} = 3.400,\ S_z = 11.05,\ S_y = 51.56,$

$\delta_{201} = 3.463,\ \delta_{003} = 1.384,\ \rho_{xz} = 0.76,\ \rho_{yz} = 0.69,$

$\delta_{220} = 4.848,\ \delta_{012} = 1.104,\ \rho_{xy} = 0.48,\ C_z = 0.23,$

$\delta_{120} = 0.689,\ \delta_{030} = 0.700,\ C_x = 0.33,\ C_y = 1.76,$

$\delta_{004} = 5.791,\ \delta_{022} = 3.400,\ \delta_{021} = 0.902,\ S_x = 7.25.$

Data set 3 (Source: Sarndal et al. (1992))**.**

$y = 1985$ population (in thousands).

$x =$ Revenues from the 1985 municipal taxation (in millions of kronor).

$z =$ Number of conservative seats in municipal council.

The data statistics are

$\delta_{400} = 88.92,\ \delta_{300} = 8.22,\ \bar{X} = 245.088,\ \bar{Y} = 29.36,$

$\delta_{210} = 7.87,\ \delta_{102} = 1.53,\ \bar{Z} = 9.10,\ N = 284,$

$\delta_{202} = 15.15,\ \delta_{040} = 88.88,\ S_z = 4.94,\ S_x = 596.33,$

$\delta_{201} = 3.42,\ \delta_{003} = 1.24,\ \rho_{xz} = 0.52,\ \rho_{yz} = 0.60,$

$\delta_{220} = 78.79,\quad \delta_{012} = 1.36,\quad \rho_{xy} = 0.961,\quad S_y = 51.56,$

$\delta_{120} = 8.02,\quad \delta_{030} = 8.77,\ C_x = 2.43,\ C_y = 1.76,$

$\delta_{004} = 5.36,\quad \delta_{022} = 12.35,\ \delta_{021} = 3.14,\ C_z = 0.54.$

Using above data statistics, MSE value of all estimators for different sample sizes are given in Tables 2, 3 and 4, respectively.

Table 2. MSE values of different estimators with respect to t_1 under different sample sizes for dataset 1

MSE	$n = 8$	$n = 12$	$n = 16$	$n = 20$
$MSE(t_1)$	0.0153	0.0088	0.0055	0.0036
$MSE(t_2)$	0.0332	0.0191	0.0120	0.0078
$MSE(t_3)$	0.0504	0.0281	0.0183	0.0118
$MSE(t_4)$	0.0013	0.0008	0.0005	0.0004
$MSE(t_5)$	0.0014	0.0008	0.0005	0.0004
$MSE(t_6)$	0.0104	0.0060	0.0038	0.0024
$MSE(t_7)$	0.0656	0.0377	0.0237	0.0154
$MSE(t_8)$	0.1083	0.0622	0.0392	0.0254
$MSE(t_{prop})$	**0.0012**	**0.0007**	**0.0004**	**0.0003**
$MSE(t_{prop1})$	0.0013	0.0007	0.0005	0.0004
$MSE(t_{prop2})$	0.0150	0.0086	0.0054	0.0035
$MSE(t_{prop3})$	0.0036	0.0015	0.0006	0.0004
$MSE(t_{prop4})$	0.0053	0.0031	0.0019	0.0012
$MSE(t_{prop5})$	0.0157	0.0090	0.0057	0.0037
$MSE(t_{prop6})$	0.0524	0.0301	0.0181	0.0123
$MSE(t_{prop7})$	0.0263	0.0132	0.0068	0.0032
$MSE(t_{prop8})$	0.0312	0.01792	0.0113	0.00731
$MSE(t_{prop9})$	0.0037	0.0016	0.0006	0.0004
$MSE(t_{prop10})$	0.0258	0.0129	0.0066	0.0030
$MSE(t_{prop11})$	0.0091	0.0032	0.0009	0.0004
$MSE(t_{prop12})$	0.0119	0.0056	0.0025	0.0009
$MSE(t_{prop13})$	0.0055	0.0032	0.0011	0.0013
$MSE(t_{prop14})$	0.0307	0.0176	0.0111	0.0072
$MSE(t_{prop15})$	0.0120	0.0057	0.0026	0.0010

Table 3. MSE values of different estimators with respect to t_1 under different sample sizes for dataset 2

MSE	$n = 35$	$n = 40$	$n = 45$	$n = 50$
$MSE(t_1)$	0.8233	0.7059	0.6146	0.5416
$MSE(t_2)$	0.8238	0.7064	0.6150	0.5419
$MSE(t_3)$	0.8992	0.7701	0.6713	0.5915
$MSE(t_4)$	0.8273	0.7093	0.6176	0.5442
$MSE(t_5)$	0.6075	0.5208	0.4535	0.3996
$MSE(t_6)$	0.6691	0.5737	0.4995	0.4402
$MSE(t_7)$	0.8184	0.7017	0.61099	0.5384
$MSE(t_8)$	1.0300	0.8832	0.7681	0.6776
$MSE(t_{prop})$	**0.4709**	**0.4037**	**0.3515**	**0.3098**
$MSE(t_{prop1})$	0.7044	0.6040	0.5259	0.4634
$MSE(t_{prop2})$	0.9638	0.8264	0.7195	0.6340
$MSE(t_{prop3})$	0.7982	0.6819	0.5914	0.5191
$MSE(t_{prop4})$	0.8160	0.6997	0.6092	0.5368
$MSE(t_{prop5})$	0.7087	0.6077	0.5291	0.4662
$MSE(t_{prop6})$	1.0258	0.8795	0.7658	0.6748
$MSE(t_{prop7})$	0.8272	0.7062	0.6121	0.5369
$MSE(t_{prop8})$	0.8492	0.7281	0.6339	0.5586
$MSE(t_{prop9})$	0.6972	0.5978	0.5205	0.4587
$MSE(t_{prop10})$	0.9813	0.8409	0.7316	0.6442
$MSE(t_{prop11})$	0.8016	0.6843	0.5932	0.5203
$MSE(t_{prop12})$	0.8213	0.7040	0.6128	0.5398
$MSE(t_{prop13})$	0.6973	0.5979	0.5206	0.4587
$MSE(t_{prop14})$	0.9855	0.8450	0.7357	0.6483
$MSE(t_{prop15})$	0.8034	0.6861	0.5949	0.5219

Table 4. MSE values of different estimators with respect to t_1 under different sample sizes for dataset 3

MSE	$n = 35$	$n = 40$	$n = 45$	$n = 50$
$MSE(t_1)$	0.8233	0.7059	0.6146	0.5416
$MSE(t_2)$	0.4354	0.3733	0.3250	0.2864
$MSE(t_3)$	3.7968	3.2555	2.8345	2.4977
$MSE(t_4)$	0.2559	0.2195	0.1911	0.1684
$MSE(t_5)$	0.5933	0.5087	0.4429	0.3903
$MSE(t_6)$	0.1842	0.1579	0.1375	0.1212
$MSE(t_7)$	0.5091	0.4365	0.3801	0.3349
$MSE(t_8)$	5.2165	4.4728	3.8943	3.4316
$\boldsymbol{MSE(t_{prop})}$	**0.1733**	**0.1486**	**0.1294**	**0.1131**
$MSE(t_{prop1})$	0.3231	0.2770	0.2412	0.2125
$MSE(t_{prop2})$	0.5292	0.4537	0.3950	0.3481
$MSE(t_{prop3})$	0.3981	0.3394	0.2937	0.2571
$MSE(t_{prop4})$	0.4123	0.3535	0.3078	0.2712
$MSE(t_{prop5})$	1.3206	1.1323	0.9859	0.8687
$MSE(t_{prop6})$	1.6841	1.4448	1.2579	1.1084
$MSE(t_{prop7})$	1.4637	1.2514	1.0864	0.9543
$MSE(t_{prop8})$	1.4890	1.2767	1.1116	0.9795
$MSE(t_{prop9})$	0.6269	0.5283	0.4517	0.3906
$MSE(t_{prop10})$	0.9010	0.7618	0.6536	0.5671
$MSE(t_{prop11})$	0.7312	0.6144	0.5237	0.4513
$MSE(t_{prop12})$	0.7479	0.6306	0.5393	0.4663
$MSE(t_{prop13})$	0.6945	0.5955	0.5185	0.4569
$MSE(t_{prop14})$	0.9798	0.8401	0.7314	0.6445
$MSE(t_{prop15})$	0.8035	0.6862	0.5941	0.5211

5. Computer Simulation

The following steps summarize the simulation procedure to find the bias and MSE of estimators.

Step 1. We select a SRSWOR of size 8 from the population 1 of size 37 (Source: Wheat Production).

Step 2. We use the data in Step 1 to find the values of estimators.

Step 3. We repeat Steps 1 and 2, 30,000 times. Thus, we obtain 30,000 values of different estimators.

Step 4. The absolute bias of the proposed estimator is obtained through the following formula:

$$AB(t_i) = \frac{1}{30000} \sum_{i=1}^{30000} | t_i - C_y |.$$

Step 5. The MSE of proposed estimator is obtained by

$$MSE(t_i) = \frac{1}{30000} \sum_{i=1}^{30000} (t_i - C_y)^2.$$

Table 5. Absolute bias and MSE values of different estimators based on simulation for Population 1

Estimator	AB	MSE
t_1	0.4661	0.2172
t_2	0.4755	0.2261
t_3	0.7250	0.5257
t_4	0.56991	0.3248
t_5	0.5317	0.2827
t_6	0.6500	0.4225
t_7	0.4638	0.2151
t_8	0.8972	0.8051
$\boldsymbol{t_{prop}}$	**0.3273**	**0.1071**
t_{prop1}	0.5502	0.3028
t_{prop2}	0.4824	0.2327
t_{prop3}	302.4952	91503.36
t_{prop4}	0.5152	0.2655
t_{prop5}	0.4503	0.2028
t_{prop6}	0.3947	0.1558
t_{prop7}	138.2517	19113.52

t_{prop8}	0.4216	0.1777
t_{prop9}	72.47301	5252.337
t_{prop10}	33.93356	1151.486
t_{prop11}	49.5978	2459.9470
t_{prop12}	49.5978	2459.9470
t_{prop13}	0.4978	0.2478
t_{prop14}	0.4364	0.1904
t_{prop15}	204.4998	41820.15

DISCUSSION AND FINDINGS

In Tables 2, 3 and 4, we have used three real life datasets to check the efficiency of our proposed class of estimators and observed that mean square error (MSEs) values of proposed family (t_{prop}) are relatively smaller than MSEs of considered estimators which are clearly indicated that the proposed estimators are more efficient. We have also proved the efficiency of our proposed class using the simulation study that is discussed in Table 5 and the results shows the supremacy of proposed class.

CONCLUSION

In this research when data on two auxiliary variables are available, a generalized family of exponential type estimators for the population mean of the study variable is developed within the parameters of a simple random sampling plan using the known coefficient of variation (CV) of study variable. The suggested estimator's properties are deduced up to the first order of approximation. Both the theoretical and empirical comparisons of the suggested estimator's efficacy are made with that of other current estimators. We also evaluated the suggested estimator's performance using data from a known natural population. Findings are shown in Tables 2-5, which demonstrates that the proposed generalized class of exponential type estimator outperforms other existing estimators by having less mean square errors. So, it is suggested the use of proposed family of estimators to estimate the population mean using the CV for more precise results.

LIMITATION AND STUDY FORWARD

No study covers all aspects of the research problem. The author should discuss the limitations or gaps of this study. And also present the future scope or plan of the study.

CONFLICT OF INTEREST AND ETHICAL STANDARDS

There exists no conflict of interest with the current organisation and no unethical practices followed during the study.

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