DEVELOPMENT OF NEW RATIO TYPE ESTIMATOR USING THE AUXILIARY INFORMATION

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ABSTRACT

In survey sampling the researchers always estimate the population mean of the study variable using the linear combination of known population values of coefficient of skewness and quartile deviation of auxiliary variable. Two modified estimators one is related to ratio and other is concerned with regression is used for estimation of population parameter of the study variable including the above linear combinations. Mean squared errors of the proposed estimators up to the first degree of approximation are obtained and compared with available estimators. The generalized proposed type estimators perform better than the competitors. Our proposed estimators give more satisfactory and efficient result as compared to all others estimators. So we can say that our proposed ratio type estimator will estimate the mean of the finite population more accurately than the others.'

Keywords: Ratio Estimator, Regression Estimator, Auxiliary , Estimator, Mean Square Error.

INTRODUCTION

In practice, it is very difficult to obtain complete information about population and therefore decisions are based on sample data for the whole population. Sampling is a technique which provide us some data from the whole lot using probability theory. Among different sampling methods, the most common and easiest method is simple random sampling (SRS) for the selection of sample. In this method each selection has equal probability without concentration of the auxiliary information. In practice with the variable of interest (Y), we collect some additional information (X) which has positive or negative correlation to the variable of interest. With the help of additional information in classical estimators we obtain more efficient results. For improving results of the estimators, many researchers are now working to use additional information. For example, Kadilar and Cingi (2006) worked on the regression type estimators.To obtain precise estimates for the population mean or total, ratio estimators are widely used. Ratio estimators are used when there is positive correlation between the auxiliary variable and variable of interest. Ratio and regression-type estimators have been appeared in large number of papers by considering different types of transformation by different researchers. The important contributions in this area is done by Kadilar and Cingi (2004), Shabbir and Gupta (2006), Khoshnevisan et al. (2007), and Koyuncu and Kadilar (2009a).

In sampling survey, researchers are working to increase the flexibility and progress the efficiency of the proposed estimators by using the auxiliary information. For example, In the current study, we have proposed an improved class of ratio and regression type estimators for estimation of mean of population. The notations used in this paper is described below

N	Population size
n	Sample size

f = $\frac{n}{N}$	Sampling fraction
Y	Study variable
X	Auxiliary variable
$\breve{X}, \breve{Y}$	Population means
$\breve{x}, \breve{y}$	Sample means
x, y	Sample totals
S_x, S_y	Population standard deviations
S_{xy}	Population covariance between variables
C_x, C_y	Population coefficient of variation
ρ	Population correlation coefficient
Md	median of auxiliary variable (X)
$b_{yx} = \frac{s_{yx}}{s_x^2}$	Sample regression coefficient of y on x
$B_{yx} = \frac{S_{YX}}{S_X^2}$	Population regression coefficient of Y on X
B(.)	Bias of the estimator
MSE(.)	Mean square error of the estimator
$\breve{Y}_i$	Existing modified ratio estimator of $\breve{Y}$
$\breve{Y}_{pr}$	Proposed modified ratio estimator of $\breve{Y}$
$QD = \frac{Q_3 - Q_1}{2}$	Quartile deviation
$\hat{R} = \frac{\breve{y}}{\breve{x}}$	Sample ratio between study and auxiliary variables
$R = \frac{\breve{Y}}{\breve{X}}$	Population ratio between study and auxiliary variables
$\beta_{1(x)}$	Population skewness
$\beta_{2(x)}$	Population kurtosis

$$\beta_{1(x)} = \frac{N\sum_{i=1}^{N}\left(X_i - \breve{X}\right)^3}{(N-1)(N-2)S_x^3}$$

$$\beta_{2(x)} = \frac{N(N+1)\sum_{i=1}^{N}\left(X_i - \breve{X}\right)^4}{(N-1)(N-2)(N-3)S_x^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$$

$$\alpha = \frac{\breve{X}}{\breve{X} + C_x}, \qquad K = \rho_{yx}\frac{C_y}{C_x}, \qquad \delta = \frac{\breve{X}}{\breve{X} + \beta_{2(x)}}, \qquad \omega_1 = \frac{\breve{X}\beta_{2(x)}}{\breve{X}\beta_{2(x)} + C_x},$$

$$\omega_2 = \frac{\breve{X}C_x}{\breve{X}C_x + \beta_{2(x)}}, \qquad \theta = \frac{\breve{X}}{\breve{X} + \left(\beta_{1(x)} - \beta_{2(x)}\right)},$$

ESTIMATORS IN LITERATURE

Section I

This section is related to ratio estimator. The literature for this section is explained as given below.
The classical ratio estimator for the population mean $\breve{Y}$ of a study variable "y" is defined as

$$\breve{y}_r = \frac{\breve{y}}{\breve{x}}\breve{X} = \hat{R}\breve{X} \tag{1}$$

$$MSE(\breve{y}_r) = \frac{(1-f)}{n}\breve{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] \tag{2}$$

Sisodia and Dwivedi [15] developed a modified ratio estimator for $\breve{Y}$ by using coefficient of variation (C_x) as the auxiliary variable and is given by

$$\breve{y}_{SD} = \breve{y}\frac{\breve{X} + C_x}{\breve{x} + C_x} \tag{3}$$

The mean square error of the above estimator upto the first order approximation is defined as

$$MSE(\breve{y}_{SD}) = \frac{(1-f)}{n}\breve{Y}^2\left[C_y^2 + C_x^2\alpha(\alpha - 2K)\right] \tag{4}$$

Singh and Kakran [18] suggested a new type of ratio estimator utilizing the coefficient of kurtosis as the auxiliary variable. The estimator is given by

$$\breve{y}_{SK} = \breve{y}\frac{\breve{X} + \beta_{2(x)}}{\breve{x} + \beta_{2(x)}} \tag{5}$$

The mean square error of the Singh and Kakran estimator up to the first order approximation is as follows:

$$MSE(\breve{y}_{SK}) = \frac{(1-f)}{n}\breve{Y}^2\left[C_y^2 + C_x^2\delta(\delta - 2K)\right] \tag{6}$$

Upadhyaya and Singh [22] presented a new modified version of the ration type estimator using two auxiliary variables that is the population correlation coefficient and coefficient of kurtosis. The estimator is given by

$$\breve{y}_{US1} = \breve{y}\frac{\breve{X}\beta_{2(x)} + C_x}{\breve{x}\beta_{2(x)} + C_x} \tag{7}$$

The mean square error of the above estimator up to first order approximation is defined as

$$MSE(\breve{y}_{US1}) = \frac{(1-f)}{n}\breve{y}^2\left[C_y^2 + C_x^2\omega_1(\omega_1 - 2K)\right] \tag{8}$$

In the same paper, Upadhyaya and Singh [22] produced another estimator using two auxiliary variables by changing the places of coefficient of variation and coefficient of kurtosis in the above estimator and is defined as

$$\breve{y}_{US2} = \breve{y}\frac{\breve{X}C_x + \beta_{2(x)}}{\breve{x}C_x + \beta_{2(x)}} \tag{9}$$

and

$$MSE(\breve{y}_{US2}) = \frac{(1-f)}{n}\breve{y}^2\left[C_y^2 + C_x^2\omega_2\left(\omega_2 - 2K\right)\right] \quad (10)$$

PROPOSED ESTIMATORS

The new estimators under simple random sampling using information on the auxiliary variables are described as below:

Let $e_\circ = \frac{\breve{y}-\breve{Y}}{\breve{Y}}$ and $e_1 = \frac{\breve{x}-\breve{X}}{\breve{X}}$, further $\breve{y} = \breve{Y}(1+e_\circ)$ and $\breve{x} = \breve{X}(1+e_1)$ are from the definition of $e_\circ$ and e_1 we derive

$$\mathrm{E}[e_\circ] = \mathrm{E}[e_1] = 0, \quad \mathrm{E}[e_\circ^2] = \frac{1-f}{n}C_y^2; \quad \mathrm{E}[e_1^2] = \frac{1-f}{n}C_x^2; \mathrm{E}[e_\circ e_1] = \frac{1-f}{n}\rho C_x C_y$$

$$\breve{y}_{p1} = \breve{y}\left[\frac{\breve{X}\frac{\beta_1(x)}{\alpha} + QD}{\breve{x}\frac{\beta_1(x)}{\alpha} + QD}\right] \quad (11)$$

where $\breve{X}_{p1} = \breve{X}\frac{\beta_1(x)}{\alpha} + QD$, $\breve{x}_{p1} = \breve{x}\frac{\beta_1(x)}{\alpha} + QD$, $\hat{R}_{p1} = \frac{\breve{y}}{\breve{x}_{p1}}$

$$\breve{y}_{p1} = \frac{\breve{y}}{\breve{x}_{p1}}\breve{X}_{p1} \quad (12)$$

solving the proposed estimator, we find:

$$\breve{y}_{p1} - \breve{Y} = \breve{Y}(e_\circ - \omega_{p1}e_1 - \omega_{p1}e_\circ e_1 + \omega_{p1}^2 e_1^2) \quad (13)$$

where $\omega_{p1} = \frac{\breve{X}\frac{\beta_1(x)}{\alpha}}{\breve{X}\frac{\beta_1(x)}{\alpha} + QD}$

So we obtained bias as:

$$Bias(\breve{y}_{p1}) = \frac{1-f}{n}\breve{Y}[\omega_{p1}^2 C_x^2 - \omega_{p1}\rho C_x C_y] \quad (14)$$

and MSE of the proposed estimator is given as:

$$MSE(\breve{y}_{p1}) = \frac{1-f}{n}\breve{Y}^2[C_y^2 + \omega_{p1}^2 C_x^2 - 2\omega_{p1}\rho C_x C_y] \quad (15)$$

THEORETICAL COMPARISON

In this section theoretical comparison are made between the existing estimators and proposed estimators of the population mean. From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators given as follows:

1. For comparison of $\breve{y}_{p1}$ and $\breve{y}_r$, $\breve{y}_{p1}$ is efficient than $\breve{y}_r$,
if $MSE(\breve{y}_{p1}) \le MSE(\breve{y}_r)$,

$$\frac{1-f}{n}\breve{Y}^2[C_y^2+\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y]\le\frac{(1-f)}{n}\breve{Y}^2\left[C_y^2+C_x^2-2\rho_{yx}C_yC_x\right]$$

$$\left(\omega_{p1}^2-1\right)C_x^2-2\left(\omega_{p1}-1\right)\rho C_xC_y\le 0 \tag{16}$$

$$\left(\omega_{p1}+1\right)C_x-2\rho C_y\le 0$$

2. For comparison of $\breve{y}_2$ and $\breve{y}_{SD}$, $\breve{y}_{p1}$ is efficient than $\breve{y}_{SD}$,
if $MSE(\breve{y}_{p1})\le MSE(\breve{y}_{SD})$,

$$\frac{1-f}{n}\breve{Y}^2[C_y^2+\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y]\le\frac{(1-f)}{n}\breve{Y}^2\left[C_y^2+C_x^2\alpha\left(\alpha-2K\right)\right]$$

$$\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y\le C_x^2\alpha\left(\alpha-2K\right)$$

$$\left(\omega_{p1}^2-\alpha^2\right)C_x^2-2\omega_{p1}\rho C_xC_y+2C_x^2\alpha\rho\frac{C_y}{C_x}\le 0$$

$$\left(\omega_{p1}+\alpha\right)C_x-2\rho C_y\le 0 \tag{17}$$

3. For comparison of $\breve{y}_{SK}$ and $\breve{y}_{p1}$, $\breve{y}_{p1}$ is efficient than $\breve{y}_{SK}$,
if $MSE(\breve{y}_{p1})\le MSE(\breve{y}_{SK})$

$$\frac{1-f}{n}\breve{Y}^2[C_y^2+\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y]\le\frac{(1-f)}{n}\breve{Y}^2\left[C_y^2+C_x^2\delta\left(\delta-2K\right)\right]$$

$$\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y\le C_x^2\delta\left(\delta-2K\right)$$

$$\left(\omega_{p1}^2-\delta^2\right)C_x^2-2\omega_{p1}\rho C_xC_y+2C_x^2\delta\rho\frac{C_y}{C_x}\le 0$$

$$\left(\omega_{p1}+\delta\right)C_x-2\rho C_y\le 0$$

4. For comparison of $\breve{y}_{US1}$ and $\breve{y}_{p1}$, $\breve{y}_{p1}$ is efficient than $\breve{y}_{US1}$,
if $MSE(\breve{y}_{p1})\le MSE(\breve{y}_{US1})$,

$$\frac{1-f}{n}\breve{Y}^2[C_y^2+\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y]\le\frac{(1-f)}{n}\breve{Y}^2\left[C_y^2+C_x^2\omega_1\left(\omega_1-2K\right)\right]$$

$$\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y\le C_x^2\omega_1\left(\omega_1-2K\right)$$

$$\left(\omega_{p1}^2-\omega_1^2\right)C_x^2-2\omega_{p1}\rho C_xC_y+2C_x^2\omega_1\rho\frac{C_y}{C_x}\le 0$$

$$\left(\omega_{p1}+\omega_1\right)C_x-2\rho C_y\le 0 \tag{18}$$

5. For comparison of $\breve{y}_{US1}$ and $\breve{y}_{p1}$, $\breve{y}_{p1}$ is efficient than $\breve{y}_{US1}$, if $MSE(\breve{y}_{p1})<MSE(\breve{y}_{US1})$,

$$\frac{1-f}{n}\breve{Y}^2[C_y^2+\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y]\le\frac{(1-f)}{n}\breve{Y}^2\left[C_y^2+C_x^2\omega_2\left(\omega_2-2K\right)\right]$$

$$\omega_{p1}^2C_x^2-2\omega_{p1}\rho C_xC_y\le C_x^2\omega_2\left(\omega_2-2K\right)$$

$$\left(\omega_{p1}^2 - \omega_2^{\,2}\right)C_x^2 - 2\omega_{p1}\rho C_x C_y + 2C_x^2\omega_2\rho\frac{C_y}{C_x} \le 0$$

$$\left(\omega_{p1} + \omega_2\right)C_x - 2\rho C_y \le 0 \tag{19}$$

Section 2

Here Regression ratio-type estimators under the same environment were proposed and compared using the same tools.

Kadilar and Cingi (2006) proposed ratio type estimators for the population mean in simple random sampling with the help of some known auxiliary information on coefficient of kurtosis and coefficient of variation. They investigated that their proposed estimators are more efficient as compared to traditional ratio estimator of the population mean.

Kadilar & Cingi (2006) estimators are given by

$$\breve{y}_1 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{\breve{x}}\breve{X} \tag{20}$$

$$\breve{y}_2 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x} + C_x)}(\breve{X} + C_x) \tag{21}$$

$$\breve{y}_3 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x} + \beta_2)}(\breve{X} + \beta_2) \tag{22}$$

The biases, related constants and the MSE for Kadilar and Cingi (2006) estimators are respectively given as follows:

$$B(\breve{y}_j) = \frac{(1-f)}{n}\breve{Y}(R_j^2 C_x^2) \tag{23}$$

$$MSE(\breve{y}_j) = \frac{(1-f)}{n}\breve{Y}^2\left[C_y^2(1-\rho_{yx}^2) + R_j^2 C_x^2\right] \tag{24}$$

where j = 1,2,3, $R_1 = 1$, $R_2 = \frac{\breve{X}}{(\breve{X} + C_x)}$, $R_3 = \frac{\breve{X}}{(\breve{X} + \beta_2)}$

Kadilar and Cingi (2004) also proposed some modified ratio estimators with the help of known value of correlation coefficient, kurtosis and coefficient of variation as follows:

$$\breve{y}_4 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x} + \rho)}(\breve{X} + \rho) \tag{25}$$

$$\breve{y}_5 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x}C_x + \rho)}(\breve{X}C_x + \rho) \tag{26}$$

$$\breve{y}_6 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x}\rho + C_x)}(\breve{X}\rho + C_x) \tag{27}$$

$$\breve{y}_7 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x}\beta_2 + \rho)}(\breve{X}\beta_2 + \rho) \tag{28}$$

$$\breve{y}_8 = \frac{\breve{y} + b(\breve{X} - \breve{x})}{(\breve{x}\rho + \beta_2)}(\breve{X}\rho + \beta_2) \tag{29}$$

The biases, related constants and the MSE for Kadilar and Cingi (2004) estimators are respectively given by

$$B(\breve{y}_j) = \frac{(1-f)}{n}\breve{Y}\,(R_j^2 C_x^2) \tag{30}$$

$$MSE(\breve{y}_j) = \frac{(1-f)}{n}\breve{Y}^2\left[C_y^2(1-\rho_{yx}^2) + R_j^2 C_x^2\right] \tag{31}$$

where j = 4, 5, 6, 7, 8

$$R_4 = \frac{\breve{X}}{\breve{X}+\rho},\; R_5 = \frac{\breve{X}C_x}{\breve{X}C_x+\rho},\; R_6 = \frac{\breve{X}\rho}{\breve{X}\rho + C_x}, R_7 = \frac{\breve{X}\beta_2}{\breve{X}\beta_2+\rho}, R_8 = \frac{\breve{X}\rho}{(\breve{X}\rho+\beta_2)}$$

PROPOSED ESTIMATOR

The proposed estimators were explained in this section, and the comparison tools were also calculated. The procedure is given below:

Let $e_\circ = \frac{\breve{y}-\breve{Y}}{\breve{Y}}$ and $e_1 = \frac{\breve{x}-\breve{X}}{\breve{X}}$, further $\breve{y} = \breve{Y}(1+e_\circ)$ and $\breve{x} = \breve{X}(1+e_1)$ are from the definition of $e_\circ$ and e_1 we derive

$$\mathrm{E}[e_\circ] = \mathrm{E}[e_1] = 0,\quad \mathrm{E}[e_\circ^2] = \frac{1-f}{n}C_y^2;\quad \mathrm{E}[e_1^2] = \frac{1-f}{n}C_x^2;\quad \mathrm{E}[e_\circ e_1] = \frac{1-f}{n}\rho C_x C_y$$

The proposed estimator $\breve{y}_{p2}$ in the form of $e_\circ$ and e_1 is given below:

$$\breve{y}_{p2} = \frac{\breve{y} + b(\breve{X}-\breve{x})}{\breve{x}\frac{\beta_1(x)}{\alpha}+QD}(\breve{X}\frac{\beta_1(x)}{\alpha}+QD) \tag{32}$$

solving the proposed estimator, we find:

where $\omega_{p2} = \frac{\breve{X}\frac{\beta_1(x)}{\alpha}}{(\breve{X}\frac{\beta_1(x)}{\alpha}+QD)}$

So we obtain Bias as:

$$Bias(\breve{y}_{p2}) = \frac{1-f}{n}(R_{p2}^2\frac{S_x^2}{\breve{Y}}) \tag{33}$$

and MSE of the proposed estimator is given as:

$$MSE(\breve{y}_{p2}) = \frac{1-f}{n}\breve{Y}^2\left[w_{p2}^2 C_x^2 + C_y^2\left(1-\rho_{xy}^2\right)\right] \tag{34}$$

where $\rho_{xy}^2 C_y C_x = \frac{S_{yx}}{\breve{Y}\breve{X}}, \frac{1}{\breve{X}} = \frac{C_x}{S_x}, B = \frac{S_{yx}}{S_x^2}$

and $R_{p2} = \frac{\breve{Y}\frac{\beta_1(x)}{\alpha}}{(\breve{X}\frac{\beta_1(x)}{\alpha}+QD)}$

THEORETICAL COMPARISON

Comparison of between existing estimators and different proposed estimators are given below

For comparison of $\breve{y}_j$ and $\breve{y}_{p2}$, $\breve{y}_{p2}$ is efficient than $\breve{y}_j$,

if $MSE(\breve{y}_{p2}) \le MSE(\breve{y}_j)$

$$\frac{1-f}{n}\breve{Y}^2\left[w_{p2}^2C_x^2+C_y^2\left(1-\rho_{xy}^2\right)\right]\leq\frac{(1-f)}{n}\breve{Y}^2\left[C_y^2(1-\rho_{yx}^2)+R_j^2C_x^2\right]$$
$$\left(w_{p2}^2-R_j^2\right)C_x^2\leq 0$$
$$\left(w_{p2}^2-R_j^2\right)\leq 0 \tag{35}$$

NUMERICAL ILLUSTRATION

To observe the merits of the proposed estimators over its opponent's numerically, we have viewed two herbal populations. Population 1 from Singh and Chaudhary (1986) and population 2 from Cochran (1977) are taken. The data given in Table 1 the population parameters and the constants were calculated. The constants are given in the next two tables. Similarly, the selection criteria for checking the performance of the new methods are given in tables 4-7. From the last two tables i.e.6,7, it showed that the proposed ratio estimator $\breve{y}_{p1}$ have least biases as compared to the existing ratio estimators $\breve{y}_i, i=1,2,3,4,5$ given in Section 1 and the bias of the suggested ratio estimator $\breve{y}_{p2}$ is less than the biases of the existing modified ratio estimators $\breve{y}_i, i=6,7,8,9,10,11$ given in Section 2. Similarly from the values of Table 6 and Table 7, it is showed that the mean squared error of the suggested ratio estimator $\breve{y}_{p1}$ is least than the mean squared errors of the existing ratio estimators $\breve{y}_i, i=1,2,3,4,5$ given in Section 1 and the mean squared error of the suggested ratio estimator $\breve{y}_{p2}$ is lesser than the mean squared errors of the existing ratio estimators $\breve{y}_i, i=6,7,8,9,10,11$ given in Section 2.

Table 1. Parameters and constants of two populations

Notations	Population 1	Population 2
N	22	49
n	5	20
$\breve{Y}$	22.6209	116.1633
$\breve{X}$	1467.5455	98.6735
ρ	0.9022	0.6904
S_y	33.0469	98.8286
C_y	1.4609	0.8508
S_x	2562.1449	102.9709
C_x	1.7459	1.0436
$\beta_2(x)$	13.3694	5.9878
$\beta_1(x)$	3.3914	2.4224
Md	534.5	64
QD	1035	78.50

Table 2|Mean square errors of section 1 existing and proposed estimator**.**

Estimator	Population 1	Population 2
$\breve{y}_r$	45.87684	232.2501
$\breve{y}_{SD}$	45.73678	228.3407
$\breve{y}_{SK}$	44.82982	212.0423
$\breve{y}_{US1}$	45.86632	231.5848
$\breve{y}_{US2}$	45.27001	212.7862
$\breve{y}_{p1}$ ($\gamma=1.5,2$)	**31.4116, 31.96963**	**154.8349, 150.7076**

Table 3| Mean square errors of section 2 existing and proposed estimator**.**

Estimator	Population 1	Population 2
$\breve{y}_1$	272.4529	584.9594
$\breve{y}_2$	271.8804	575.9043
$\breve{y}_3$	268.1202	536.6219
$\breve{y}_4$	272.1568	578.9111
$\breve{y}_5$	272.2833	579.1613
$\breve{y}_6$	271.8185	571.9903
$\breve{y}_7$	272.4308	583.9405
$\breve{y}_8$	267.6576	517.7825
$\breve{y}_{p2}$ $(\gamma = 2,3,4)$	**196.6675, 196.6675, 196.6675**	**308.4952, 260.4151, 231.3125**

The above tables shows MSEs of for section 1 and section 2 of the proposed and existing estimators for two types of data. We see that the MSEs of the proposed estimators for different values of γ are less than the MSE of the existing estimators. So we can say that both the proposed estimators are better than the existing estimators.

Table 4|Theoretical conditions results for section 1

Conditions	Population 1 $(\gamma = 1.5, 2)$		Population 2 $(\gamma = 1.5, 2)$	
1	-1.627204	-1.627204	-0.227598	-0.218548
2	-1.626146	-1.626146	-0.226171	-0217122
3	-1.619167	-1.619167	-0.219801	-0.210751
4	-1.627124	-1.627124	-0.227357	-0.218308
5	-1.622583	-1.622583	-0.220108	-0.211059

Table 5|Theoretical conditions results for section 2

Conditions	Population 1 $(\gamma = 2,3,4)$			Population 2 $(\gamma = 2,3,4)$		
1	-0.501197	-0.620772	-0.701987	-0.635712	-0.746270	-0.813189
2	-0.498822	-0.618397	-0.699612	-0.614891	-0.725448	-0.792368
3	-0.483223	-0.602798	-0.684013	-0.524563	-0.635120	-0.702040
4	-0.499969	-0.619544	-0.700759	-0.621805	-0.732362	-0.799282
5	-0.500493	-0.620068	-0.701283	-0.622380	-0.732937	-0.799857
6	-0.498565	-0.618140	-0.699355	-0.605891	-0.716448	-0.783368
7	-0.501105	-0.620680	-0.701895	-0.633369	-0.743927	-0.810846
8	-0.481304	-0.600879	-0.682094	-0.481243	-0.591801	-0.658720

CONCLUSION

This paper examines two types of estimators using auxiliary information. One is related to ratio type estimator and another one is with regression type mentioned in section 1 and 2 respectively. To assess the performance of both estimators we derived theoretical conditions and used two types of data in support of the results. The results for the MSE are evaluated for the proposed as well as other type of estimators. The MSE of two estimators for both the data is found smaller than the existing estimators.

Therefore, under the above mentioned situations our proposed estimators give more satisfactory and efficient result as compared to all others estimators. So we can say that our proposed estimators will estimate the mean of the finite population more accurately than the others.

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