International Journal of Contemporary Issues in Social Sciences
Volume 3, Issue 3, 2024 ISSN(P):2959-3808 | 2959-2461

AN IMPROVED ROBUST REGRESSION TYPE MEAN ESTIMATOR USING RE-DESCENDING M-ESTIMATOR UNDER TWO-STAGE SAMPLING APPROACH

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Received: July 15, 2024 **Revised:** August 15, 2024 **Accepted:** August 30, 2024 **Published:** September 12, 2024

ABSTRACT

When the data includes some outliers, estimating the population average using the ordinary least squares (OLS) approach is not effective. In this article, we develop a novel technique for computing the population average using regression robust-type estimator in two phase sampling. The developed technique uses a robust regression type estimator called re-descending M-estimation. The mean square error (MSE) equation is generated using a first-order approximation and examined with existing estimating techniques in order to evaluate the efficacy of the new approach. Additionally, the proposed estimator's percentage relative efficiency (PRE) is determined compared to other estimators. The effectiveness of the proposed method is demonstrated using real data sets. According to the results, the developed estimator performs efficient than other existing estimators in the literature.

Keywords: Outliers, M-estimator, Two stage sampling, Robust regression.

1. INTRODUCTION

The OLS technique is highly sensitive to outliers and hence does not yield effective outcomes when there are any. Robust regression-type utilizing re-descending M-estimator is used as an alternative technique to obtaining accurate estimates when information contains outliers. Several robust regression-type techniques are documented in literature however Ref [1] M-estimation methods, which eliminates remove the impact of outliers, is the most commonly used technique.

In survey sampling, there are several ways we can improve our estimate by using supplementary data. It should be noted that the ratio, regression, and product type estimation techniques are useful when the supplementary data are available Ref [2]. However, situations can also occur where many authors develop various estimators using auxiliary data, improving the performance of the estimation methods. In this situation, numerous writers, including Ref [3], constructed a number of improved and modified estimators using auxiliary data.

Consider the finite population $S = \{x_1, x_2, \ldots, x_N\}$ of size N. A first stage sample of size $n_1 (n_1 < N)$ is utilized to observe the supplementary information only while both research and supplementary information's are studied on the second stage sample of size $n_2 (n_2 < N)$. The sample means for Y and X are denoted by $\bar{y}$ and $\bar{x}$, respectively. We supposed to be the mean of the sample of X for the first stage is $\bar{x}_1$, and sample means of Y and X for the second stage are $\bar{y}_2$ and $\bar{x}_2$ respectively. At both stages the sample is selected by simple random sampling without replacement (SRSWOR).

Typically, two stage sampling is normally used to get additional data regarding population parameters. Two phase sampling was first introduced by Ref [4] to get the data on strata sizes. Supplementary variables was incorporated in a thoughtful sampling techniques by (Ref [5-14]) in order to obtain efficient estimates. Ref [15] suggested a novel family of robust regression in ratio-type estimation methods by utilizing Huber MM, LTS, and LMS estimators. Ref [16] suggested efficient estimation method of population parameter utilizing robust regression-type estimators in two phase sampling. Ref [17] developed a novel class of robust regression type estimation methods for mean estimation, under simple and two-stage random sampling techniques with additional data is available. Ref [18] presented a robust quantile regression-type mean estimate technique using two additional information. A compromised-imputation and EWMA based memory-type mean estimation methods using quantile regression were proposed by Ref [19]. Ref [20] suggested a novel methods for estimating countable population variance utilizing ranks of additional information in two stage sampling.

The article's remaining sections are arranged as follows: provide some existing estimators in two stage sampling in Section 2. In Section 3, the suggested estimator are given. In Section 4, a numerical analysis is carried out. The article concludes at Section 5.

2. Some Existing Estimators in Two-stage Sampling

2.1 Ref [21] developed estimators

Ref [21] suggested the estimation methods $\bar{y}_{ols_l}$. According to their analysis, the developed estimation methods outperform then OLS estimation method in terms of effectiveness. Following Ref [21] developed following regression-in-ratio estimation methods for population parameter in two stage sampling.

$$\bar{y}_{ols_l} = \frac{\bar{y}_2 + b(\bar{x}_1 - \bar{x}_2)}{\Omega_l \bar{x}_2 + \theta_l}(\Omega_l \bar{x}_1 + \theta_l) \qquad l = 1,2,3,4,5 \tag{1}$$

where $b = \frac{s_{yx}}{s_x^2}$ is calculated by OLS estimator of regression coefficient.

$$\Omega_1 = 1 \ \& \ \theta_1 = 0, \Omega_2 = 1 \ \& \ \theta_2 = C_x, \Omega_3 = 1 \ \& \ \theta_3 = \beta_2(x), \Omega_4 = \beta_2(x) \ \& \ \theta_4 = C_x, \\ \Omega_5 = C_x \quad \& \quad \theta_5 = \beta_2(x)$$

Where $\beta_2(x)$ and C_x are the coefficient of kurtosis and coefficient of variation (CV) of additional information respectively. The MSE estimators in Eq (1) is given as

$$MSE(\bar{y}_{ols_l}) = \bar{Y}^2 \lambda_2 C_y^2 + \{B\bar{X} + \bar{Y}(1-\delta_l)\}(\lambda_2 - \lambda_1) C_x^2 \{B\bar{X} + \bar{Y}(1-\delta_l) - 2\bar{Y}H_{yx}\} \tag{2}$$

where

$$\delta_1 = 0 \ , \delta_2 = \frac{C_x}{\bar{X}} \ , \delta_3 = \frac{\beta_2(x)}{\bar{X}} \ , \delta_4 = \frac{C_x}{\bar{X}\beta_2(x)} \ , \delta_5 = \frac{\beta_2(x)}{\bar{X}C_x} \tag{3}$$

It's crucial to keep in mind that $E(b) = B$.

2.2 Ref [8] suggested estimators

Ref [8] suggested the following robust estimation methods to estimate the population parameter in two stage sampling in case of outliers by utilizing data on the supplementary variable's using Ref [1] M-estimator.

$$\bar{y}_{rob_l} = \frac{\bar{y}_2 + b_{rob}(\bar{x}_1 - \bar{x}_2)}{\Omega_l \bar{x}_2 + \theta_l}(\Omega_l \bar{x}_1 + \theta_l) \tag{4}$$

Where b_{rob} is generated using the robust regression Huber M-estimator with $l = 1,2,3,4 \ \& \ 5$.

using $y_i = a + bx_i + r_i$, Ref [1] created the following objective function for robust regression-type as follows.

$$\rho_2(r) = \begin{cases} \frac{r^2}{2} & |r| \le v \\ v|r| - \frac{r^2}{2} & |r| > v \end{cases} \tag{5}$$

Huber (1964) created setting $v = 1.5s$, where "s" is the estimate of population standard deviation (SD) of error (residuals) terms. The b_{rob} is gained by decreasing the $\sum_{i=1}^{n} \rho_2(y_i - a - bx_i)$ by utilizing the objective function provided in Eq (5). However, the MSE expression provided by Ref [8] contains some writing errors, the true expression for the MSE of the estimator given in Eq (4) is as follows

$$MSE\left(\bar{y}_{rob_l}\right) = \bar{Y}^2\lambda_2 C_y^2 + \{B_{rob}\bar{X} + \bar{Y}(1-\delta_l)\}(\lambda_2-\lambda_1)C_x^2\{B_{rob}\bar{X} + \bar{Y}(1-\delta_l) - 2\bar{Y}H_{yx}\} \quad (6)$$

The values of δ_l are similar as given in (3)

For more details see Ref [8].

2.3 Ref [16] suggested estimators

Ref [16] developed regression-type ratio estimation methods for population parameter in two-stage sampling based on a novel proposed robust-type re-descending M-estimator (RM). The RM are stated by:

$$\bar{y}_{RM_l} = \frac{\bar{y}_2 + b_{RM}(\bar{x}_1 - \bar{x}_2)}{\Omega_l\bar{x}_2 + \theta_l}(\Omega_l\bar{x}_1 + \theta_l) \quad (7)$$

where b_{RM} is provided by Ref [16]. The Raza's objective function's $\rho_1(r_l)$ design is explained as

$$\rho_3(r_i) = \frac{v^2}{2a}\left\{1 - \left[1 + \left(\frac{r}{v}\right)^2\right]^{-a}\right\} \; for \; |r| \geq 0$$

where "v" and "a" are tuning parameters. The optimal tuning constant values for the current investigation are $v = 1.5$ and a are $(5 \; and \; 7)$. The b_{RM} the RM is utilized in the MSE equation of the regression in ratio estimation methods in Eq (7).

$$MSE\left(\bar{y}_{RM_l}\right) = \bar{Y}^2\lambda_2 C_y^2 + \{B_{RM}\bar{X} + \bar{Y}(1-\delta_l)\}(\lambda_2-\lambda_1)C_x^2\{B_{RM}\bar{X} + \bar{Y}(1-\delta_l) - 2\bar{Y}H_{yx}\} \quad (8)$$

For more details see Ref [16].

3. Proposed Estimator

By extending the idea of existing estimator, we propose the robust regression mean estimator in two-phase sampling in Eq (9) as

$$\bar{y}'_N = k_1\{\bar{y}_2 + b_{RM}(\bar{x}_1 - \bar{x}_2)\} + k_2(\bar{x}_1 - \bar{x}_2) \quad (9)$$

We also use following notations $\xi_{y_2} = \frac{\bar{y}_2 - \bar{Y}}{\bar{Y}}, \xi_{x_1} = \frac{\bar{x}_1 - \bar{X}}{\bar{X}} \; and \; \xi_{x_2} = \frac{\bar{x}_2 - \bar{X}}{\bar{X}}$.

Employing these terms, we can write $E\left(\xi_{y_2}\right) = E\left(\xi_{x_1}\right) = E\left(\xi_{x_2}\right) = 0, E\left(\xi_{y_2}^2\right) = \lambda_2 C_y^2, E\left(\xi_{x_1}^2\right) = \lambda_1 C_x^2, E\left(\xi_{x_2}^2\right) = \lambda_2 C_x^2, E\left(\xi_{y_2}\xi_{x_1}\right) = \lambda_1 C_{yx}, E\left(\xi_{y_2}\xi_{x_2}\right) = \lambda_2 C_{yx}, and \; E\left(\xi_{x_1}\xi_{x_2}\right) = \lambda_1 C_x^2$. Now, expending $\bar{y}'_N$ in terms of $\xi's$ as given below:

$$\bar{y}'_N = k_1\{\bar{Y}(1+\xi_{y_2}) + b_{RM}\bar{X}(\xi_{x_1} - \xi_{x_2})\} + k_2\bar{X}(\xi_{x_1} - \xi_{x_2})$$

$$\bar{y}'_N - \bar{Y} = k_1\{\bar{Y}(1+\xi_{y_2}) + b_{RM}\bar{X}(\xi_{x_1} - \xi_{x_2})\} + k_2\bar{X}(\xi_{x_1} - \xi_{x_2}) - \bar{Y} \quad (10)$$

Squaring Eq (10), we may obtain the MSE of the estimator $\bar{y}'_N$ upto first-order approximation by applying expectation.

$$MSE(\bar{y}'_N) = \bar{Y}^2 + k_1^2\vartheta_{A_N} + k_2^2\vartheta_{B_N} + 2k_1k_2\vartheta_{C_N} - 2k_1\vartheta_{D_N} \quad (11)$$

where

$$\vartheta_{A_N} = \left[\bar{Y}^2(1+\lambda_2 C_y^2) + (\lambda_2-\lambda_1)B_{RM}\bar{X}\{B_{RM}\bar{X}C_x^2 - 2\bar{Y}C_{yx}\}\right],$$

$$\vartheta_{B_N} = \bar{X}^2(\lambda_2-\lambda_1)C_x^2$$

$$\vartheta_{C_N} = (\lambda_1-\lambda_2)\bar{X}\left[\bar{Y}C_{yx} - B_{RM}\bar{X}C_x^2\right]$$

$$\vartheta_{D_N} = \bar{Y}^2$$

Which is minimum for

$$k_{1(opt)} = \left[\frac{\vartheta_{B_N}\vartheta_{D_N}}{\vartheta_{A_N}\vartheta_{B_N} - \vartheta_{C_N}^2}\right]$$

and

$$k_{2(opt)} = \left[-\frac{\vartheta_{C_N}\vartheta_{D_N}}{\vartheta_{A_N}\vartheta_{B_N} - \vartheta_{C_N}^2}\right]$$

Substituting these values in Eq (11), we have

$$MSE(\bar{y}'_N)_{opt} = \left[\bar{Y}^2 - \frac{\vartheta_{B_N}\vartheta_{D_N}^2}{\vartheta_{A_N}\vartheta_{B_N} - \vartheta_{C_N}^2}\right]$$

$\lambda_1 = \left(\frac{1}{n_1} - \frac{1}{N}\right)$ and $\lambda_2 = \left(\frac{1}{n_2} - \frac{1}{N}\right)$

4. Numerical Study

This section compares the effectiveness of the suggested estimators to the existing estimation methods in terms of MSE's and PRE's using two real-world datasets. PRE of an estimator can be computed through the following expressions:

$$PRE(\bar{y}_Q, \bar{y}'_N) = \frac{\bar{y}_Q}{\bar{y}'_N} \times 100$$

where $Q = \bar{y}_{ols_l}, \bar{y}_{rob_l}\ and\ \bar{y}_{RM_l}$ $\quad l = 1,2,3,4,5$

4.1. Population 1

To illustrate the effectiveness of the suggested estimation method in this article, we analyze a data-set of apples utilized in Ref [16]. For population 1, we take the following variables into account: The quantity of apple-fruits produced during 1999 is expressed by y, while x is the list of apple-fruit trees during 1999.

Table 1: Statistics Regarding Population 1

$\boldsymbol{N = 94}$	$S_x = 1607.573$
$\boldsymbol{\rho = 0.9011}$	$S_y = 29907.48$
$\boldsymbol{\bar{X} = 724.099}$	$\beta_2(x) = 27.703$
$\boldsymbol{\bar{Y} = 9384.309}$	$B_{RM} = 7.0560$
$\boldsymbol{C_x = 2.220}$	$B = 16.764$
$\boldsymbol{S_{yx} = 43324466}$	$B_{rob} = 12.287$

The scatter plots in Figure 1 clearly shows outlier values, indicating that the data are appropriate for our suggested estimators.

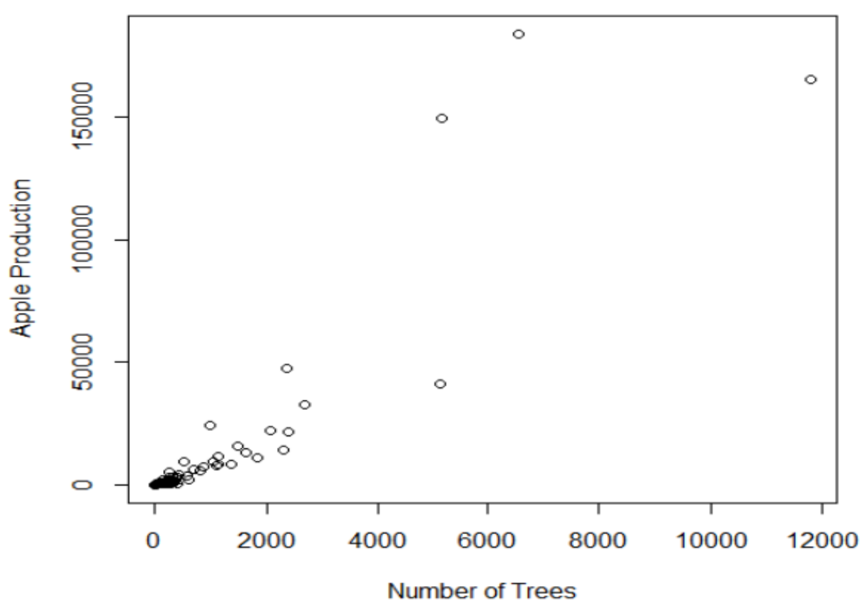

Fig1. Apple production and number of trees

4.2. Population 2

To illustrate the effectiveness of the suggested estimation method in this article, we analyze a time series data of Australian Relative Wool Prices (ARWP) used in Ref [16]. For population 2, we take the following variables into account: Time is taken as independent variable (x) and weekly wool price is taken as study variable (y).

Table 2: Statistics Regarding Population 2	
$\boldsymbol{N = 321}$	$S_x = 94.49718$
$\boldsymbol{\rho = 0.702458}$	$S_y = 0.1612209$
$\boldsymbol{\bar{X} = 161.8567}$	$\beta_2(x) = 27.703$
$\boldsymbol{\bar{Y} = 0.2580692}$	$B_{RM} = 0.0006625$
$\boldsymbol{C_x = 0.5838324}$	$B = 0.0011984$
$\boldsymbol{S_{yx} = 10.70189}$	$B_{rob} = 0.0008528$

The scatter plots in Figure 2, clearly shows outlier values, indicating that the data are appropriate for our suggested estimators.

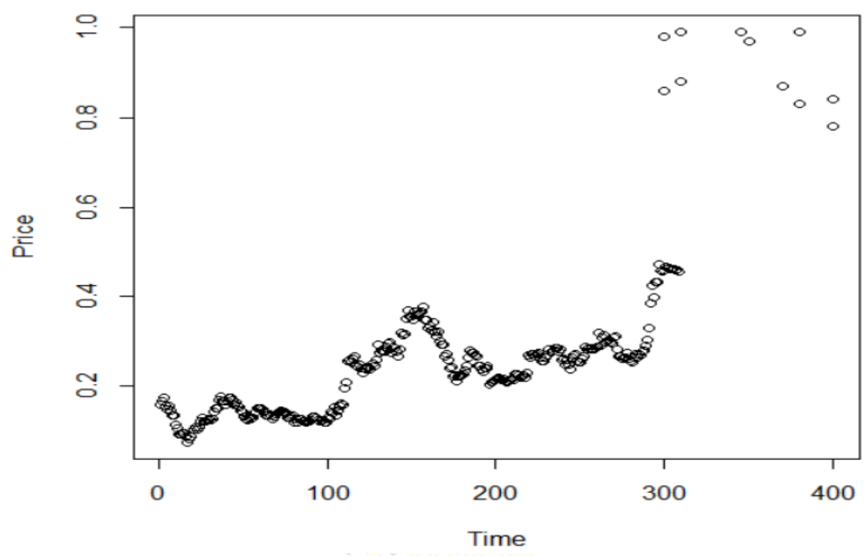

Fig 2: Price of wool and time with 3% outliers.

Table 3: MSE of existing and proposed estimators in Two Phase Sampling at $n_1 = 50$ for Populaion 1			
Estimators	$n_2 = 5$	$n_2 = 10$	$n_2 = 25$
$\bar{y}_{ols_1}$	116756266	56543795	20416313
$\bar{y}_{ols_2}$	116278006	56331235	20363173
$\bar{y}_{ols_3}$	110893323	53938043	19764875
$\bar{y}_{ols_4}$	116738977	56536111	20414392
$\bar{y}_{ols_5}$	114086994	55357452	20119727
$\bar{y}_{rob_1}$	72108349	36700277	15455433
$\bar{y}_{rob_2}$	71795571	36561264	15420680
$\bar{y}_{rob_3}$	68310427	35012311	15033442
$\bar{y}_{rob_4}$	72097033	36695247	15454176
$\bar{y}_{rob_5}$	70369267	35927351	15262202
$\bar{y}_{RM_1}$	43561479	24012779	12283559
$\bar{y}_{RM_2}$	43442053	23959700	12270289
$\bar{y}_{RM_3}$	42176363	23397172	12129657
$\bar{y}_{RM_4}$	43557143	24010852	12283077
$\bar{y}_{RM_5}$	42909246	23722897	12211088
$\bar{y}'_N$	26851958	17488090	10355699

Table 4: PRE of proposed estimator's w.r.t existing estimators in Two Phase Sampling at $n_1 = 50$ for Population-1

Estimators	$n_2 = 5$	$n_2 = 10$	$n_2 = 25$
	$\bar{y}'_N$	$\bar{y}'_N$	$\bar{y}'_N$
$\bar{y}_{ols_1}$	434.8147	323.3274	197.1505
$\bar{y}_{ols_2}$	433.0336	322.112	196.6374
$\bar{y}_{ols_3}$	412.9804	308.4273	190.8599
$\bar{y}_{ols_4}$	434.7503	323.2835	197.132
$\bar{y}_{ols_5}$	424.874	316.5437	194.2865
$\bar{y}_{rob_1}$	268.5404	209.8587	149.2457
$\bar{y}_{rob_2}$	267.3756	209.0638	148.9101
$\bar{y}_{rob_3}$	254.3965	200.2066	145.1707
$\bar{y}_{rob_4}$	268.4982	209.8299	149.2335
$\bar{y}_{rob_5}$	262.0638	205.439	147.3797
$\bar{y}_{RM_1}$	162.2283	137.3093	118.6164
$\bar{y}_{RM_2}$	161.7836	137.0058	118.4883
$\bar{y}_{RM_3}$	157.07	133.7892	117.1303
$\bar{y}_{RM_4}$	162.2122	137.2983	118.6118
$\bar{y}_{RM_5}$	159.7993	135.6517	117.9166

Table 5: MSE of existing and proposed estimators in Two Phase Sampling at $n_1 = 200$ for Populaion 2

Estimators	$n_2 = 25$	$n_2 = 50$	$n_2 = 100$
$\bar{y}_{ols_1}$	0.001304297	0.0005869777	0.0002283181
$\bar{y}_{ols_2}$	0.001298575	0.0005845256	0.0002275008
$\bar{y}_{ols_3}$	0.001055599	0.0004803929	0.0001927899
$\bar{y}_{ols_4}$	0.00130409	0.000586889	0.0002282886
$\bar{y}_{ols_5}$	0.0009067406	0.0004165965	0.0001715244
$\bar{y}_{rob_1}$	0.0009971977	0.0004553638	0.0001844468
$\bar{y}_{rob_2}$	0.0009927187	0.0004534442	0.000183807
$\bar{y}_{rob_3}$	0.0008074536	0.0003740449	0.0001573405
$\bar{y}_{rob_4}$	0.0009970357	0.0004552944	0.0001844237
$\bar{y}_{rob_5}$	0.0007006187	0.0003282585	0.0001420784
$\bar{y}_{RM_1}$	0.0008599712	0.0003965524	0.0001648431
$\bar{y}_{RM_2}$	0.0008561763	0.000394926	0.0001643009
$\bar{y}_{RM_3}$	0.0007026891	0.0003291458	0.0001423742
$\bar{y}_{RM_4}$	0.0008598338	0.0003964936	0.0001648234
$\bar{y}_{RM_5}$	0.0006189938	0.0002932764	0.0001304177
$\bar{y}'_N$	0.0005059399	0.0002455757	0.0001146228

Table 6: PRE of proposed estimator's w.r.t existing estimators in Two Phase Sampling at $n_1 = 200$ for Population-2

Estimators	$n_2 = 25$	$n_2 = 50$	$n_2 = 100$
	$\bar{y}'_N$	$\bar{y}'_N$	$\bar{y}'_N$
$\bar{y}_{ols_1}$	257.7968	239.0211	199.1908
$\bar{y}_{ols_2}$	256.6659	238.0226	198.4778

$\bar{y}_{ols_3}$	208.6412	195.6191	168.195
$\bar{y}_{ols_4}$	257.7559	238.985	199.1651
$\bar{y}_{ols_5}$	179.219	169.6408	149.6425
$\bar{y}_{rob_1}$	197.0981	185.4271	160.9164
$\bar{y}_{rob_2}$	196.2128	184.6454	160.3581
$\bar{y}_{rob_3}$	159.5948	152.3135	137.2681
$\bar{y}_{rob_4}$	197.066	185.3988	160.8962
$\bar{y}_{rob_5}$	138.4787	133.669	123.953
$\bar{y}_{pro_1}$	169.975	161.4787	143.8135
$\bar{y}_{RM_1}$	169.2249	160.8164	143.3405
$\bar{y}_{RM_2}$	138.8879	134.0303	124.211
$\bar{y}_{RM_3}$	169.9478	161.4547	143.7964
$\bar{y}_{RM_4}$	122.3453	119.4241	113.7799

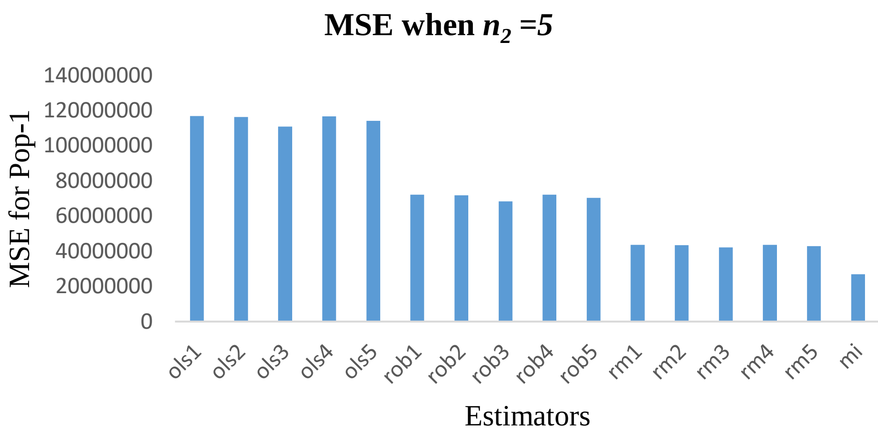

Fig 3: MSE for Pop-1 when $n_2 = 5$.

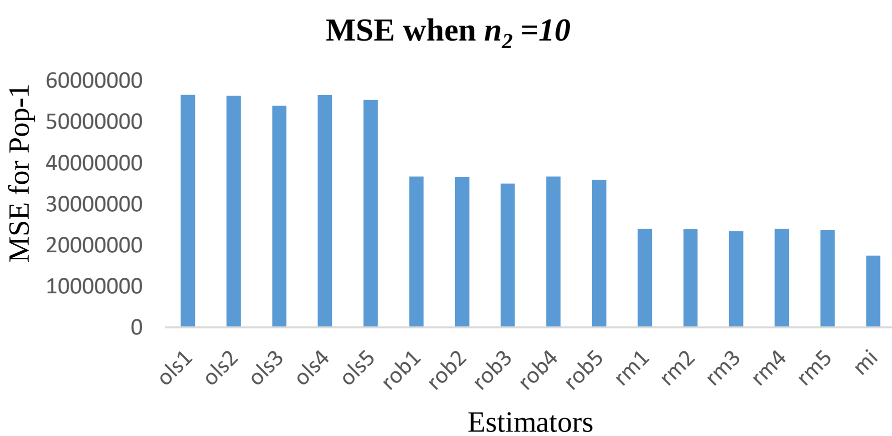

Fig 4: MSE for Pop-1 when $n_2 = 10$.

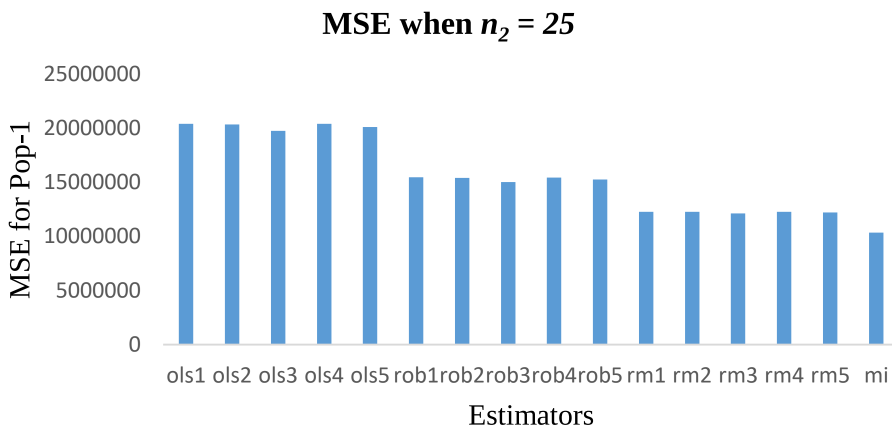

Fig 5: MSE for Pop-1 when $n_2 = 25$.

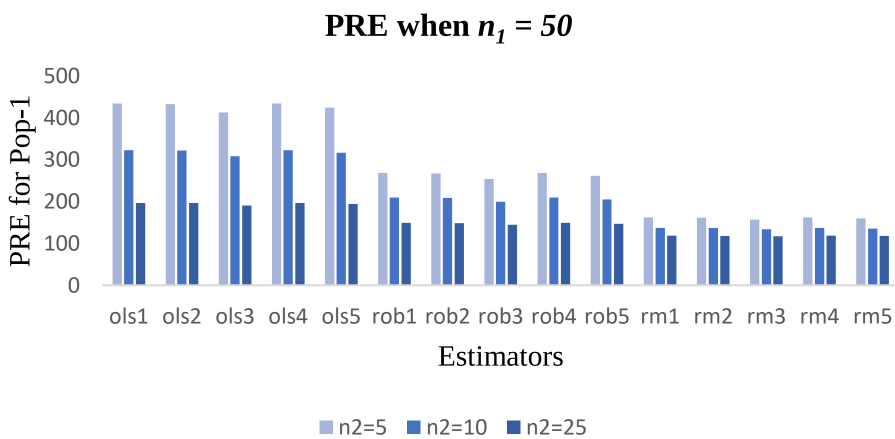

Fig 6: PRE for Pop-1 when $n_1 = 50$

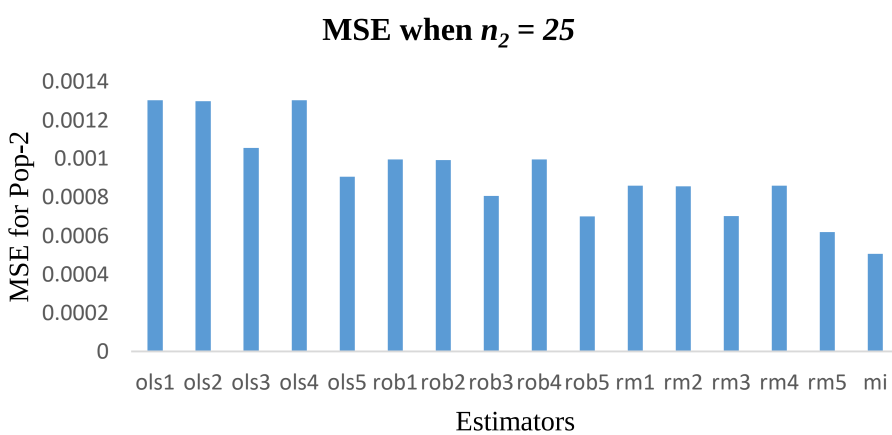

Fig 7: MSE for Pop-2 when $n_2 = 25$.

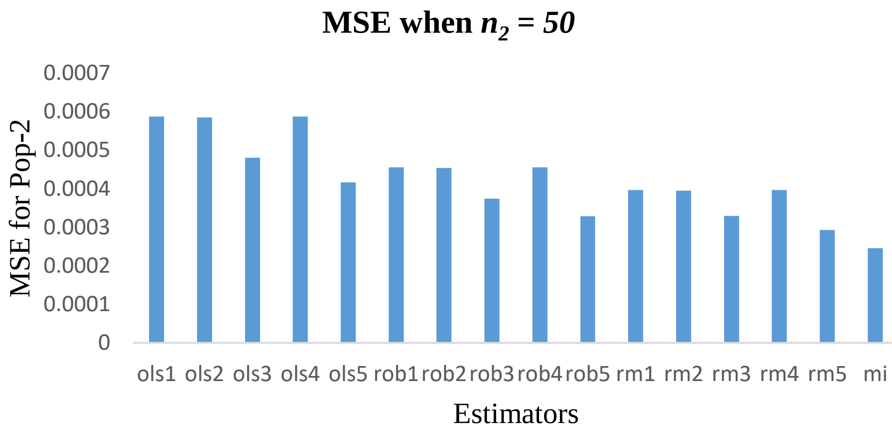

Fig 8: MSE for Pop-2 when $n_2 = 50$.

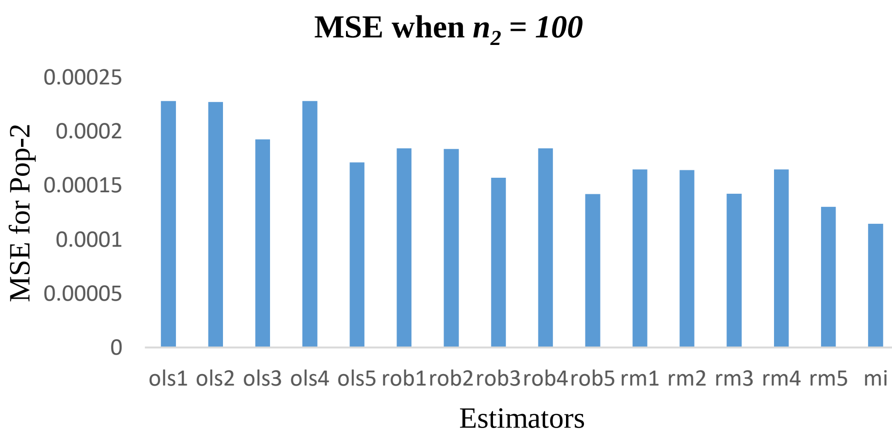

Fig 9: MSE for Pop-2 when $n_2 = 100$.

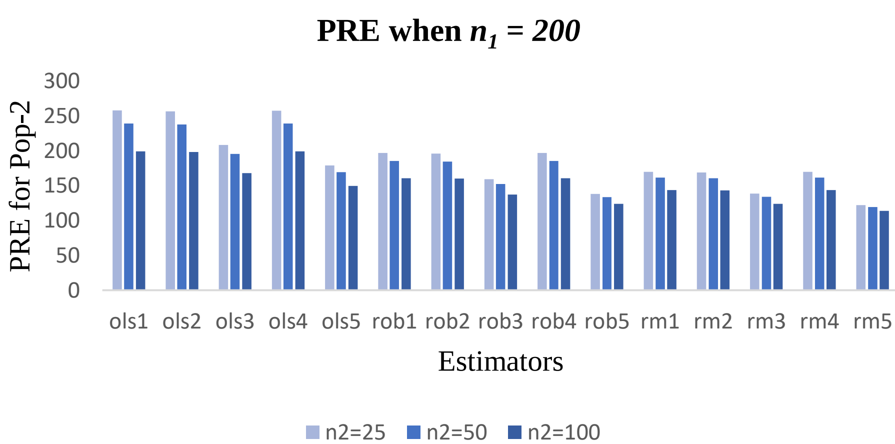

Fig10: PRE for Pop-2 when $n_1 = 200$

4.3. Interpretation.

The interpretation is provided in the following upcoming points:

1. Estimators $(\bar{y}_{rob_1}, \dots, \bar{y}_{rob_5})$ are performing better than $(\bar{y}_{ols_1}, \dots, \bar{y}_{ols_5})$.
2. Estimators $(\bar{y}_{RM_1}, \dots, \bar{y}_{RM_5})$ is performing better than $(\bar{y}_{ols_1}, \dots, \bar{y}_{ols_5})$.
3. Estimator $\bar{y}'_N$ is performing better than $(\bar{y}_{ols_1}, \dots, \bar{y}_{ols_5})$, $(\bar{y}_{rob_1}, \dots, \bar{y}_{rob_5})$ and $(\bar{y}_{RM_1}, \dots, \bar{y}_{RM_5})$.
4. The visual representation of MSE's results are in provided in Figures 3-10.

Thus, it can be seen from Tables 3, 4, 5 and 6 that the suggested estimators are more effective than the existing estimation methods under the two stage sampling for the provided datasets.

5. Conclusion

The OLS technique is ineffective for predicting the population mean when the data contains some outliers. In Two stage sampling, under the decided circumstances, using the RM, a new robust regression type estimator has been proposed. Utilizing the numerical illustration it has also been demonstrated that the proposed estimator produces smallest MSE value as compared to the existing estimators. When data contain outliers, it is found that the suggested regression-type mean estimator performs better than the existing estimators. This study is just the beginning; there is an enormous opportunity ahead for developing improved estimating techniques for many different types of data under different sampling techniques.

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