

UNVEILING EXCHANGE RATE DYNAMICS: A MARKOV CHAIN MODELING OF PAKISTAN'S CURRENCY TRENDS AND PERSISTENCE PATTERNS

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ABSTRACT

In Pakistan, we fitted an advanced Markov chain model based on the daily and weekly currency rates. Based on each of these price indices, a sequence of daily exchange rates for prices (over the previous day) was calculated for 2282 days between January 2016 and March 2022. The estimated period was split into two 2282-day halves: pre-liberalization and post-liberalization, where return periods for these three exchange rate states, stable state probabilities, and state transition probabilities were estimated. Thus, the long-run probability (after the memory of the initial state is lost in the process of continuous transitions) of a positive currency, change (i.e., currency increasing) in the case of the daily and weekly exchange rate in any day in the period of January 2016 is 0.411. Hence the return period of inflation is $1/0.411 = 2.44$ days. This means that the pre-liberalization period witnessed on pre-dollar exchange rate increasing (as measured over the previous days) once on 2.44 days or about 3 times in 20 days. In Pakistan, it was discovered that the status of the exchange rate generally endures with a high likelihood and a significantly shorter return period. This suggests a high and affected exchange rate.

INTRODUCTION

The exchange rate, which is the cost of a currency relative to several other currencies, is among the most crucial topics in global politics and finance. Many emerging economies have used the exchange rate as a tool to manage capital and trade flows. The prices of various assets can be affected by both direct and indirect changes in the exchange rate. Exchange rates play an important role in a country's economy, especially in international trade. Therefore, it is important to be able to predict their future value, fluctuations, and stability in the face of changing economic factors. Exchange rate models also consider institutional setups, corporate technologies, and the relative cost of households in a country, in

addition to taxes and tariffs imposed by two nations. Nonlinear methods like machine learning [1, 2] and random walks [3, 4] are used in recent publications on exchange rate forecasting and prediction. For instance, Ranjit et al. [5] employ artificial neural networks (ANN) and recurrent neural networks (RNN) in machine learning to create a prediction model for Nepalese rupees against the euro, pound sterling, and US dollar. Ca'Zorzi et al. studied the regularities in foreign exchange markets in industrialized countries with flexible regimes using panel data approaches and nonlinear models. found in Wang et al [4]. In their research, the nonlinear smooth transition regression (STR) method was used

to model and forecast the exchange rate. Due to the exchange rate's central significance in economic growth and the desire for recognition from the outside world, it has been extensively discussed in literature. [6][7][8][9].

Any country's exchange rate demonstrates how competitive it is in the global economy and enhances its internal stability [10]. Exchange rate volatility is primarily caused by under- and overestimation of domestic currency. It illustrates how the indigenous currency compares to other currencies. [11]. Since the end of the Bretton-Woods framework, exchange rates have been discovered to be very unstable. Due to their relevance on a worldwide scale and impact on economic stability, productivity, trade patterns, and investments, exchange rates have been a topic of discussion for many scholars. [12][13][14] [15]. Studies have been done to predict what will happen with stable and unstable exchange rates. Exchange rate stability led to increased foreign investments, increased exports, and a favorable shift in the nation's trade balance [16][17][18][19][20]. Unpredictability in the evolution of exchange rates boosts inflation, widens the trade imbalance, and reduces investment [21]

The exchange rate in Pakistan has sparked off a large number of studies, specifically in the cause-effect framework, with an implicit, assumption on the persistence of the cumulative dynamics of the currency rises. However, no study has come up to model these dynamics and verifies the assumption. Therefore, the current study aims to investigate Pakistan's exchange rate's propensity for self-accelerating currency increases. The persistence of the cumulative dynamics of currency is measured in terms of long-run probability currency increases, estimated by modeling a Markov chain of the daily and weekly movement of currency in Pakistan. During the last 2282 days from 1st January 2016 to 31st March, 2022. More specifically, In the framework of the Markov chain model, we analyze the behavior of the successive daily and weekly changes in a selected set of exchange rates to find whether the changes are positive (tentatively suggesting that the rate of currency is increasing), negative (rate of currency is decreasing), or zero (currency is stable). We next estimate the transition probabilities, from which we calculate the steady state probability for these three states and evaluate the transition of the states. The model for each currency is specified separately for the two periods

under study, 1st January 2016 (pre-liberalization) and 31st March 2022 (post-liberalization), to find out whether the currency dynamics have been more accelerating (i.e, with higher probability) during the transition period, as is commonly held.

This study is divided into four sections, including this introduction. The next section provides the theoretical framework of the Markov chain model concerning the time series on currency movements, and section third the results from the model on the currency changes in Pakistan. The last section briefly summarizes and conclusion of the study.

Exchange rates are calculated using a variety of methods and ideas by economists and scholars. None of the models offered the precise parameters to anticipate how currency rates would behave in all circumstances but taken collectively, they help assist and decrease this unpredictability by pinpointing the crucial factors that influence exchange rates in various scenarios. The connection between a country's commerce and exchange rates is a common theme in literature. Siddiqui et al [22] investigated Pakistan's exchange rate behavior from 1960 to 1994 using a two-stage least squares method. The analysis concludes that trade openness promotes exchange rate appreciation. However, the findings show that terms of trade had little impact on the real exchange rate.

Ye et al [23] present two novel forecasting models, the Markov switching monetary model (MSMM) and the Markov switching random walk model (MSRW). in addition to the conventional monetary model and the random walk model using the Markov switching technique. The research indicates that the two hybrid models outperform the standard models in forecasting the exchange rate.

For three daily exchange rate series, March et al. [24] developed two-state Markov models. They next examine the stability of implementing the predictions by comparing the results to the performance of straightforward chartist trading rules. The results show that (i) Markov models model the data well, (ii) the effectiveness of once-pro-table trading rules has deteriorated significantly since the 1990s, and (iii) in their current state, Markov models are unstable and inappropriate for predicting.

Using time-series data from 1972 to 2013, Raza et al [8]. investigate the factors influencing Pakistan's exchange rate. The outcomes of the Gregory and Hansen structural break co-integration approach, the autoregressive-distributed lag bound testing co-

integration approach, and the Johansen and Juselius co-integration approach all support the existence of a significant long-run relationship between a few of the variables under consideration. The long-run estimates show a strong inverse relationship between exchange rates and trade terms, trade openness, and economic growth. Exchange rates, however, are positively and significantly impacted by the money supply and inflation rate. The Granger causality test demonstrates that there is a causal link between Pakistan's exchange rate and economic development and trade openness. Contrarily, the one-way causation runs from the explanatory variable to the country's inflation rate, money supply, and terms of trade. The results of the current analysis may help decision-makers choose the most effective and least volatile exchange rate for Pakistan to achieve long-term sustainable economic growth.

Using the three consumer price indices, the general wholesale pricing indices, and a chosen set of sectional wholesale price indices across 240 months between April 1981 and March 2001, PILLALI et al. [25] built a Markov chain model for price variations in India. The estimation period has been divided into two parts, pre-liberalization, and post-liberalization, each lasting ten years. We consider three general states of monthly price changes: positive (tentatively indicating inflation), negative (deflation), and zero (stable prices), and we calculate the probabilities of transition between these three states of price level changes as well as their steady states and return periods. It is shown that there is a substantial possibility of inflation in India continuing and that the recovery time would be much shorter. This implies that price rises have been gradual, having an inflationary effect as a result.

Method and Material

For the period from 1st January 2016 to March 31st, 2022, Secondary data will be obtained from ExchangeRate.org.uk. The model for daily and weekly exchange rates is specified separately for the two periods under study from 1st January 2016 to 18 August 2018 (pre-liberalization) and 19 August 2018 to 31 March 2022 (post-liberalization).

In addition to the previously mentioned theoretical framework and model specification, this section also discusses the estimation of the parameters of the models to be used from the available data and an analysis strategy that provides a detailed description

of the data used and applications of the method to address the study's objectives.

Stochastic Processes

A set of random variables make up a stochastic process, $X_t: t \in T$, where T denotes the parameter space and S denotes the state space of the process. Both continuous and discrete zones can exist in S and T . As a result, four different types of stochastic processes may be discussed, depending on the kind of space. Discrete state-space processes are taken into consideration in this study. With $\{X(t): t \in T\}$ and $\{X_t: t \in T\}$ denoting processes with continuous and discrete parameter space, respectively.

Suppose that $t_0, t_1 \in T; t_0 < t_1$.

Then, the function,

$$F(X_0, X_1, t_0, t_1) = P[X(t_1) \leq x_1 | X(t_0) \leq x_0], \quad (1)$$

is referred to as the stochastic process' conditional distribution function $\{X(t): t \in T\}$.

about a procedure with a discrete parameter space. $\{X(t): t \in T\}$,

There are

$$P_{ij}^{(m,n)} = (X_n = j | X_m = i), \quad (2)$$

where the parameter space is $m, n \in T$ and the state space is $i, j \in S$. Transition probabilities are the probability in (1) and (2).

Markov chain

Markov dependency is present in the stochastic processes $X(t): t \in T$ and $X_n: n \in T$, respectively.

$$P[X(t) \leq x | X(t_n) = x_n, X(t_{n-1}) = X_{n-1}, \dots, X(t_0) = x_0] = P[X(t) \leq x | X(t_n) = x_n], \quad (4)$$

$$P[X_n = j | X_{n1} = i_1, X_{n2} = i_2, \dots, X_{nk} = i_k] = P[X_n = j | X_{n1} \in i_1], \quad (5)$$

Markov processes, or Markov chains, are discrete state space stochastic processes that meet (4) and (5).

Markov chain

The stochastic process known as the Markov chain is named after Andrey Markov. Andrey Markov is a mathematician from Russia. He used mathematics to study poetry and a brand-new area of probability that is today referred to as the Markov chain. According to Peach et al., the Markov chain is one of the finest approaches for forecasting. Numerous academics have used Markov chains to make future predictions

in a variety of fields, including PageRank, gambling, and the birth-death cycle, among others. (Levin and others, 2009). We will use the Markov chain process to predict the daily and weekly exchange rates of rupees as compared to other currencies like dollars and euros in Pakistan.

Two-State MC Model

The occurrence of rise and drop may be predicted using a straightforward two-state (increase and decrease) first-order Markov chain model.

The probability of transitioning between states is what gives rise to the state Markov chain model (Liven, 2009; Serfozo, 2009). The transitional probability of switching from an increase state to a decrease state or from a decrease state to an increase state is defined by Equation 1. The probability of a two-state Markov chain transition is displayed in the table below.

2×2 Transition probability matrixes			
		Coming	
		Decrease	Increase
Current	Decrease	P_{11}	P_{12}
	Increase	P_{21}	P_{22}

Three-State MC model

The occurrence of rise, decrease, or stability may be predicted using a straightforward first-order Markov chain model with three states (increase, decrease, and stable). The transition from increase to decrease, from decrease to stable, and stability to increase is depicted in the table below.

When seen as a three-state Markov chain with state spaces $S = 1, 2$, and 3 , the data collected over the course of the days may be categorized as decreasing, increasing, and stable. The transition probability is defined as the ratio of the current (day) exchange rate to the prior (day) exchange rate.

$$P = P_{ij} = P\left(\frac{j}{i}\right) \quad \text{where } i, j \in S$$

Table 3×3 Transition probability matrixes					
		Coming			
		Decrease (1)	Increase (2)	Stable (3)	Total
Current	Decrease	P_{11}	P_{12}	P_{13}	1
	Increase	P_{21}	P_{22}	P_{23}	1
	Stable	P_{31}	P_{32}	P_{33}	1

Results and discussion

In this part, we provide the findings from the Markov chain model's application to Pakistan's daily and weekly currency rates. a process involving the analysis of daily price exchange rates for various price indices over a span of 2282 days, from January 2016 to March 2022. The primary objective of this analysis is to calculate the maximum likelihood (ML) estimate of state transition probabilities within each category of price indices. We consider the temporal behavior pattern of the exchange rate in its 3 states: decreasing, increasing, and stable. Thus, assuming a simple Markov chain for the distribution of the daily exchange rate, the conditional probability that the price change is in state i (say, decrease) on any given day given that it was in state j (say, increase) on the previous day is given by $P_{ij}, i, j = \text{decrease, increase, stable}$. The probability of a price transition from one condition to another in the "previous" day to the "current" day are estimated. Assume we want to test the null hypothesis that the observed relationship is the result of a Markov chain with a transition matrix P^e of equation probabilities for each of the three states of inflation, i.e., $P^e = (P_{ij}^e, j = 1/3)$, i.e., $j = 1, 2, 3$. We have, then, using equation (7),

$$\sum_{i=1}^3 \sum_{j=1}^3 \frac{n_{ij} (\hat{P}_{ij} - \frac{1}{3})^2}{\frac{1}{3}} = 101.53$$

For 6 df, $\Pr [\chi^2 \geq 101.53]$ being extremely small, the null hypothesis probability is rejected. The same holds for the second period as well.

We may calculate the steady-state probability from the transition probabilities: Equation (13) was used to get three simultaneous equations for the pre-liberalization, and they are

$$-0.6777 \pi (1) + 0.6294 \pi (2) + 0.0482 \pi (3) = 0$$

$$0.5059 \pi (1) - 0.5277 \pi (2) + 0.0217 \pi (3) = 0$$

$$0.2000 \pi (1) + 0.3000 \pi (2) - 0.5 \pi (3) = 0$$

One of these equations, say, the first one, is then replaced, according to equation (14), by

$$\pi (1) + \pi (2) + \pi (3) = 1,$$

and the solutions obtained are:

$$\pi (1) = 0.411, \pi (2) = 0.526, \text{ and } \pi (3) = 0.062$$

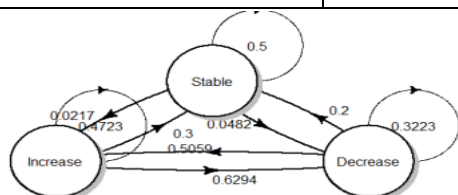
$$\text{That is, } \pi = [0.411 \ 0.526 \ 0.062]^T$$

which gives the long-run probabilities of increasing, decreasing, and stable exchange rates, based on daily and weekly data in January 2016, after a long period.

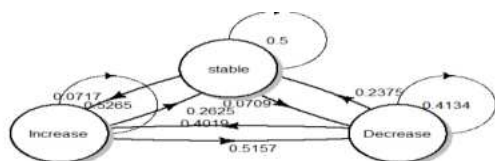
Table 1

Transition probability of exchange rates Pre-liberalizations period for daily and weekly Pre-Dollar and Pre-Euro

Transition Probability matrix for Pre dollar				
	Decrease	Increase	Stable	Total
Decrease	0.3223	0.6294	0.0482	1
Increase	0.5059	0.4723	0.0217	1
Stable	0.2000	0.3000	0.5000	1
Chi-Square Value: 101.53			P-Value:0.9075	



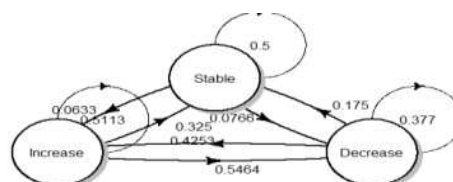
Transition Probability matrix for Pre euro				
	Decrease	Increase	Stable	Total
Decrease	0.4275	0.5318	0.0407	1
Increase	0.4163	0.5498	0.0339	1
Stable	0.2576	0.2424	0.5000	1
Chi Square Value: 92.937			P-Value:0.9203	



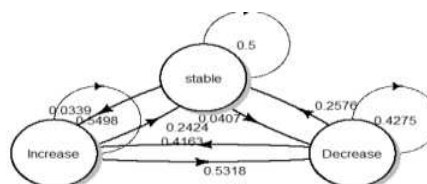
Transition Probability matrix for Pre dollar weekly			
	Decrease	Increase	Total
Decrease	0.3571	0.6429	1
Increase	0.4625	0.5375	1

Post-liberalizations period for daily and weekly Post-Dollar and Post-Euro

Transition Probability matrix for Post dollar				
	Decrease	Increase	Stable	Total
Decrease	0.3770	0.5464	0.0766	1
Increase	0.4253	0.5113	0.0633	1
Stable	0.1750	0.3250	0.5000	1
Chi-Square Value: 74.813			P-Value:0.9453	

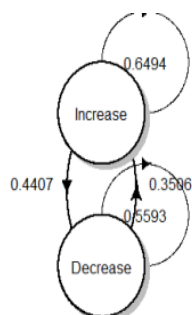


Transition Probability matrix for Post euro				
	Decrease	Increase	Stable	Total
Decrease	0.4134	0.5157	0.0709	1
Increase	0.4019	0.5265	0.0717	1
Stable	0.2375	0.2625	0.5000	1
Chi Square Value: 72.98			P-Value:0.9476	

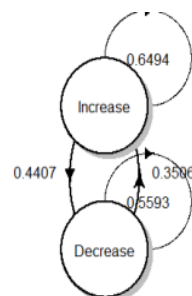


Chi Square Value: 3.5923e-32			P-Value:1	
Probability matrix for Post dollar weekly				
	Decrease	Increase	Total	
Decrease	0.6329	0.3671	1	
Increase	0.2569	0.7431	1	
Chi Square Value: 3.6057e-32			P-Value:1	

Transition Probability matrix for Pre euro weekly			
	Decrease	Increase	Total
Decrease	0.5593	0.4407	1
Increase	0.3506	0.6494	1
Chi-Square Value: 3.6161e-32			P-Value:1



Transition Probability Matrix for Post Euro Weekly			
	Decrease	Increase	Total
Decrease	0.5119	0.4881	1
Increase	0.3942	0.6058	1
Chi-Square Value: 3.6139e-32			P-Value:1



Thus, the long-run probability (after the memory of the initial state is lost in the process of continuous transitions) of a positive currency, change (i.e., currency increasing) in the case of daily and weekly exchange rate in any day in the period of January 2016 is 0.411. Hence the return period of inflation is $1/0.411 = 2.44$ days. This means that the pre-liberalization period witnessed on pre-dollar exchange rate increasing (as measured over the previous days) once on 2.44 days or about 3 times 20 days. The long-run probability of the post-dollar for the post-liberalization is found to be 0.377, with a return period of 2.65 days that is, we had increasing currency 5 times in 13 days during August 2018. In the majority of the categories in both eras, the results (Table 1) indicate that the durability of

currency exchange (increase-increase state transition) is considerably high. The transitional probability matrix shows that the exchange rate for the pre-dollar on the current day decreases, and then there is a 32.23% chance that the exchange rate on an upcoming day for the pre-dollar also decreases. The chances that the exchange rate will increase in the upcoming day are 62.94%. There are 4.82% chance that the exchange rate will remain stable in the upcoming day. Similarly, if on a given day the exchange rate increases, the chances for decreasing, increasing, and remaining stable for the exchange are 50.59%, 47.23%, and 2.17% respectively. Similarly, if on a given day the exchange rate is stable, the chances for decreasing, increasing, and remaining stable for the exchange rate are 20%, 30%, and 50% respectively.

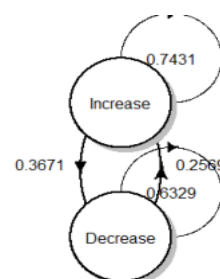
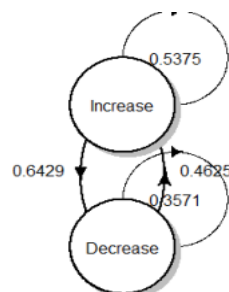


Table 2
Steady State Probability

Steady-state probability for three states						
	Pre-Liberalization			Post Liberalization		
	Decreasing	Increasing	Stable	Decreasing	Increasing	Stable
Daily dollar exchange rate	0.411	0.526	0.062	0.377	0.502	0.121
Daily Euro exchange rate	0.41	0.521	0.069	0.386	0.489	0.125

Steady-state probability for two states				
	Pre Liberalization		Post Liberalization	
	Decreasing	Increasing	Decreasing	Increasing
Weekly dollar exchange rate	0.418	0.582	0.412	0.588
Weekly Euro exchange rate	0.443	0.577	0.447	0.533

Table 3
Return Periods

Return Probability								
	Pre Liberalization				Post Liberalization			
	Decreasing	Increasing	Stable		Decreasing	Increasing	Stable	
Daily dollar exchange rate	2.44	1.90	16.12	20.46	2.65	1.99	8.26	12.9
Daily Euro exchange rate	2.44	1.92	14.49	18.85	2.59	2.04	8	12.63

Return Probability						
	Pre Liberalization			Post Liberalization		
	Decreasing	Increasing		Decreasing	Increasing	
Weekly dollar exchange rate	2.39	1.72	4.11	2.42	1.70	4.12
Weekly euro exchange rate	2.25	1.73	3.98	2.23	1.87	4.1

The above table 2 shows the long run probability of the two and three states of exchange rate. As We had a very high degree of confidence that the exchange rate would remain over time. In the long run probabilities of increasing exchange rate for daily dollar and euro exchange rate are persists and weekly dollar and weekly euro exchange rate are also persisting.

From the above table 3 show that it can be observed that the expected length of decreasing in pre daily dollar exchange rate is 2.44, increasing is 1.90 and stable is 16.12.the exchange cycle is found out to be 20.46 days. Now for the pre-daily euro the length of decreasing is 2.44, increasing 1.92 and stable is 14.49. The exchange cycle is found out to be 18.85 days and For the Post daily dollar exchange rate, the length of decreasing, increasing and stable is 2.65,

1.99, and 8.26.and the exchange cycle is found to be 12.9 days. For post-daily euro, the length of decreasing, increasing and stable is 2.59, 2.04 and 8 the exchange cycle is found out to be 12.63 days.

Conclusion

In this thesis we have developed a Markov chain model for exchange rate in Pakistan and applied it to study whether the daily exchange rate and weekly exchange rate rises were cumulating and thus capable of upbringing exchange rate in effect. The dynamics of cumulating is measured in term of long run probability of decreasing daily price change over the previous day; a significantly higher probability implies the persistence of unabated price over time that is exchange rate.

For the period from January 1st, 2016 to March 31st, 2022 Secondary data will be obtained from ExchangeRate.org.uk. The model for daily and weekly exchange rates is specified separately for the two periods under study from 1st January 2016 to 18 August 2018 (pre-liberalization) and 19 August 2018 to 31 March 2022 (post-liberalization) covering 2282 days which constitute our time series date. We have considered three general states of exchange rates decrease, increase, and stable, and estimating using ML method. The state transition probability for each of the daily and weekly exchange rates is considered. We have also attempted a statistical inference by testing the null hypothesis of equal probability for all three states. The analysis has been carried out for the two periods pre-liberalization (2016) and post-liberalization (2018). In the case of the majority of the rise indices and both eras, the short-run transition probability calculation clearly shows a widespread persistence of an increase in the exchange rate with a very high likelihood. The latter period probability is much higher. The null hypothesis of equal probability for all the state transitions is rejected in all the cases.

As we predicted, there is a very strong likelihood that all three of the exchange rate situations will remain over the long term. In the steady state, there is a great likelihood that the daily dollar and euro exchange rates will continue to rise as well as the weekly dollar and euro exchange rates.

It can be observed that the expected length of decrease in the pre-daily dollar exchange rate is 2.44, increasing is 1.90 and the stable is 16.12. the exchange cycle is found to be 20.46 days. Now for the pre-daily euro the length of decreasing is 2.44, increasing 1.92 and stable is 14.49. The exchange cycle is found to be 18.85 days and For the Post daily dollar exchange rate, the length of decreasing, increasing, and stable is 2.65, 1.99, and 8.26. and the exchange cycle is found to be 12.9 days. For the post-daily euro, the length of decreasing, increasing, and stable is 2.59, 2.04, and 8 the exchange cycle is found to be 12.63 days.

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Competing Interest

The authors declare no competing interest in the content relevant to this article.

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